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ADAPTIVE CONSTANT MODULUS ALGORITHM BASED ON COMPLEX GIVENS ROTATIONS

A. Boudjellal¹ K. Abed-Meraim¹ A. Belouchrani² Ph. Ravier¹

¹ Polytech Orleans, Prisme Laboratory, France; ² Ecole Nationale Polytechnique, Algiers, Algeria.

ABSTRACT

This paper deals with adaptive Constant Modulus Algorithm (CMA) for the blind separation of communication signals. Ikhlef et al. proposed in 2010 an efficient block implementation of the CMA using Givens rotations. We introduce herein a fast adaptive implementation of this method which exploits recent developments on whitening techniques together with appropriate updating of the used statistics and efficient selection of the Givens rotation parameters. The proposed algorithm shows significantly improved performance with respect to existing techniques as illustrated by the simulation results.

Index Terms— Constant Modulus Algorithm (CMA), Adaptive Whitening, Complex Givens Rotations.

1. INTRODUCTION

The Constant Modulus Algorithm is one of the most efficient techniques for blind equalization and blind separation of communication signals and has therefore attracted a lot of interest in the literature, e.g. [5, 6, 10, 11] for the blind equalization and [7, 8, 9] for the blind source separation. All these methods improve and extend the original version introduced more than three decades ago in [3, 4].

In particular, several adaptive algorithms have been proposed for the optimization of the Constant Modulus cost function including the adaptive Analytical CMA (ACMA) in [9] considered as one of the most efficient CMA implementations and used below in our comparative study.

Herein, we consider another type of adaptive implementations using unitary Givens rotations. The proposed algorithm has the advantage of fast convergence and improved separation quality for a moderate computational cost with respect to the methods in [8, 9]. Below we provide the data model and a description of the algorithm's development.

2. PROBLEM FORMULATION

The instantaneous mixture \mathbf{x}_t of d transmitted sources \mathbf{s}_t received through an m -antenna array is modeled as follows:

$$\mathbf{x}_t = \mathbf{A}\mathbf{s}_t + \mathbf{n}_t \quad (1)$$

where \mathbf{A} is the $m \times d$ mixing matrix and \mathbf{n}_t is an additive noise of covariance $\sigma_n^2 \mathbf{I}$. By stacking T samples of the received data in one matrix $\mathbf{X} = [\mathbf{x}_1 : \mathbf{x}_T]$, equation (1) becomes $\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}$ where $\mathbf{S} = [\mathbf{s}_1 : \mathbf{s}_T]$ and $\mathbf{N} = [\mathbf{n}_1 : \mathbf{n}_T]$. Blind Source Separation aims to recover the unknown sources \mathbf{S} from observed mixtures \mathbf{X} , relying only on some assumptions on the statistical properties of the original sources¹. This is equivalent to finding a $d \times m$ separation matrix \mathbf{W} that leads to the estimated source vector (up to scaling and permutation ambiguities [14]): i.e. $\mathbf{Z} = \mathbf{W}\mathbf{X} = \hat{\mathbf{S}}$.

3. GIVENS CONSTANT MODULUS ALGORITHM

Originally, the CMA in [3, 4] was designed in such a way it exploits the fact that the transmitted sources are generated from a finite alphabet having a constant modulus² R and try to restore this property by minimizing the deviation of the restored signals modulus from this constant. This leads to the following Constant Modulus Criterion (CMC):

$$\mathcal{J}(\mathbf{W}) = \sum_{i=1}^m \sum_{j=1}^T \left(|z_{ij}|^2 - R \right)^2 \quad (2)$$

where z_{ij} is the $(i, j)^{th}$ entry of the output matrix \mathbf{Z} . The minimization of (2) has led to a large number of algorithms belonging to the CMA class. In particular, the authors in [1] proposed a two-step iterative Jacobi-like algorithm for the minimization of the CMC after data pre-whitening. The bloc version of the GCMA, presented in [1], estimates the separation matrix \mathbf{W} as a product of a $m \times d$ whitening matrix \mathbf{B} and a $d \times d$ unitary matrix \mathbf{U} : $\mathbf{W} = \mathbf{U}\mathbf{B}$. First the data bloc \mathbf{X} is whitened by applying the matrix \mathbf{B} , estimated using any whitening algorithm e.g. [1, 9]. Then, the unitary matrix \mathbf{U} is estimated as the minimizer of (2). As presented in [1], an iterative algorithm is used to minimize (2) where \mathbf{U} is rewritten as a product of complex Givens rotations. In other words,

¹Standard hypotheses consist of assuming that: (i) The mixing matrix \mathbf{A} is a tall full column rank matrix ($m \geq d$), (ii) The original sources are mutually independent, (iii) The additive noise is white, Gaussian, and independent from the source signals.

²It has been shown later that the CMA can be applied for any sub-Gaussian sources [13]. Also, because of the scaling ambiguity, one can chose $R = 1$, without loss of generality.

at iteration k , the current estimate of the separation matrix $\mathbf{W}^{(k-1)}$ is updated as $\mathbf{W}^{(k)} = \mathbf{G}^{(k)}(p, q, \theta, \alpha) \mathbf{W}^{(k-1)}$, where $\mathbf{G}^{(k)}(p, q, \theta, \alpha)$ is the complex unitary Givens matrix with diagonal elements equal to one except for the two elements $g_{pp} = g_{qq} = \cos(\theta)$ and its off-diagonal elements are null except for the elements $g_{pq} = -g_{qp}^* = e^{i\alpha} \sin(\theta)$. The optimization of the CMC in (2) w.r.t. the rotation parameters is equivalent to the minimization of:

$$\mathcal{J}(\theta, \alpha) = 2(\mathbf{u}^T \mathbf{Q} \mathbf{u}) + \lambda(\mathbf{u}^T \mathbf{u} - 1) \quad (3)$$

where λ is the Lagrange multiplier, $\mathbf{Q} = \sum_{t=1}^T \mathbf{r}_t \mathbf{r}_t^T$, $\mathbf{u} = [\cos(2\theta), \sin(2\theta)\cos(\alpha), \sin(2\theta)\sin(\alpha)]^T$, and

$$\mathbf{r}_t = \left[\frac{1}{2} \left(\left| z_{pt}^{(k-1)} \right|^2 - \left| z_{qt}^{(k-1)} \right|^2 \right), \mathcal{R} \left(z_{pt}^{(k-1)} z_{qt}^{(k-1)*} \right), \mathcal{I} \left(z_{pt}^{(k-1)} z_{qt}^{(k-1)*} \right) \right]^T$$

The couple of angles (θ, α) minimizing the quadratic form in (3) under the constraint $\mathbf{u}^T \mathbf{u} = 1$ is chosen such that:

$$\cos(\theta) = \sqrt{\frac{u_1 + 1}{2}} \quad \text{and} \quad e^{i\alpha} \sin(\theta) = \frac{u_2 + iu_3}{\sqrt{2(u_1 + 1)}} \quad (4)$$

where $\mathbf{u} = [u_1, u_2, u_3]^T$ is the unit-norm least eigenvector of \mathbf{Q} associated to its smallest eigenvalue.

4. ADAPTIVE GCMA

We propose here an efficient adaptive GCMA algorithm which proceeds at each time instant t to the following steps.

4.1. Step 1: Adaptive whitening

The adaptive whitening is based on the fast subspace tracking algorithm GOPAST in [12] where the whitening matrix is computed according to as follows:

$$\mathbf{B}^{(t)} = \mathbf{\Lambda}_t^{-\frac{1}{2}} \mathbf{V}_t^H \quad (5)$$

with \mathbf{V}_t and $\mathbf{\Lambda}_t$ are adaptive estimates of the d principal eigenvectors and their corresponding eigenvalue diagonal matrix of the data covariance matrix \mathbf{R}_x , respectively. The numerical complexity of this adaptive whitening algorithm is equal to $4md + O(d^2)$ flops at each time instant t (see [12] for details). The updated matrix is then applied to the current data vector according to:

$$\bar{\mathbf{y}}_t = \mathbf{B}^{(t)} \mathbf{x}_t \quad (6)$$

4.2. Step 2: Adaptive estimation of the unitary factor

The algorithm consists of applying, at each time t , a Givens rotation $\mathbf{G}^{(t)}(p, q, \theta, \alpha)$ to update the unitary matrix \mathbf{U} : i.e.

$$\mathbf{U}^{(t)} = \mathbf{G}^{(t)}(p, q, \theta, \alpha) \mathbf{U}^{(t-1)} \quad (7)$$

The angles θ and α are computed such that the following adaptive constant modulus criterion is minimized ($0 < \beta < 1$ being a forgetting factor):

$$\mathcal{H}(\mathbf{G}^{(t)}, t) = \sum_{k=1}^t \beta^{t-k} \sum_{i=1}^m \left(|z_{ik}|^2 - R \right)^2 \quad (8)$$

Now, if one applies the Givens rotation to the last data vector, only the last term in (8) would contribute to the estimation of the angle parameters. This approach led to poor estimation and tracking performance. To improve the estimation of the Givens parameters, we propose to compute the unitary transformation as if applied to all past data. After some straightforward derivations, the CMC criterion becomes:

$$\mathcal{J}(\theta, \alpha) = 2(\mathbf{u}^T \mathbf{Q}^{(t)} \mathbf{u}) + \lambda(\mathbf{u}^T \mathbf{u} - 1) \quad (9)$$

where $\mathbf{r}_t = \left[\frac{1}{2} \left(|y_{pt}|^2 - |y_{qt}|^2 \right), \mathcal{R}(y_{pt} y_{qt}^*), \mathcal{I}(y_{pt} y_{qt}^*) \right]^T$, $\mathbf{y}_t = \mathbf{U}^{(t-1)} \bar{\mathbf{y}}_t$, and

$$\mathbf{Q}^{(t)} = \sum_{k=1}^t \beta^{t-k} \mathbf{r}_k \mathbf{r}_k^T = \beta \mathbf{Q}^{(t-1)} + \mathbf{r}_t \mathbf{r}_t^T \quad (10)$$

The optimal couple of angles (θ, α) that minimizes the quadratic form in (9) under the constraint $\mathbf{u}^T \mathbf{u} = 1$ is given in (4) where $\mathbf{u} = [u_1, u_2, u_3]^T$ is the least eigenvector of matrix $\mathbf{Q}^{(t)}$ associated to its smallest eigenvalue.

Note that we have a set of $d(d-1)/2$ matrices $\mathbf{Q}^{(t)}$ for all index pairs (p, q) to be updated at each time instant. A direct updating using (10) would cost approximately $81d^2$ flops. This cost can be reduced by almost a factor of 10 by exploiting existing redundancy between these matrices. To achieve this, the following quantities have to be updated separately:

$$\left\{ \begin{array}{l} \sum_{k=1}^t \beta^{t-k} y_{pk} y_{qk}^* \\ \sum_{k=1}^t \beta^{t-k} |y_{pk}|^2 |y_{qk}|^2 \\ \sum_{k=1}^t \beta^{t-k} |y_{pk}|^2 y_{pk} y_{qk}^* \\ \sum_{k=1}^t \beta^{t-k} y_{pk}^2 y_{qk}^{2*} \end{array} \right. \quad (11)$$

and hence, four statistics are defined as follows:

$$\mathbf{S}_i^{(t)} = \sum_{k=1}^t \beta^{t-k} \mathbf{F}_{i1}(\mathbf{y}_k) \mathbf{F}_{i2}(\mathbf{y}_k)^H, \quad i : 1 : 4 \quad (12)$$

where:

$$\left\{ \begin{array}{l} \mathbf{F}_{11}(\mathbf{y}_k) = \mathbf{F}_{12}(\mathbf{y}_k) = \mathbf{F}_{32}(\mathbf{y}_k) = \mathbf{y}_k \\ \mathbf{F}_{21}(\mathbf{y}_k) = \mathbf{F}_{22}(\mathbf{y}_k) = \mathbf{y}_k \odot \mathbf{y}_k^* \\ \mathbf{F}_{31}(\mathbf{y}_k) = \mathbf{y}_k \odot \mathbf{y}_k^* \odot \mathbf{y}_k \\ \mathbf{F}_{41}(\mathbf{y}_k) = \mathbf{F}_{42}(\mathbf{y}_k) = \mathbf{y}_k \odot \mathbf{y}_k \end{array} \right. \quad (13)$$

\odot being the Hadamard product. At the acquisition of a new sample, the above statistics are updated as follows:

$$\mathbf{S}_i^{(t)} = \beta \mathbf{S}_i^{(t-1)} + \mathbf{F}_{i1}(\mathbf{y}_t) \mathbf{F}_{i2}(\mathbf{y}_t)^H, \quad i : 1 : 4 \quad (14)$$

Initialization:	$\mathbf{U}^{(0)} = \mathbf{I}_d$
For $t = 1, 2, \dots$ do	
Compute	\mathbf{A}_t and \mathbf{V}_t using GOPAST [12]
Whitening	$\bar{\mathbf{y}}_t = \mathbf{B}^{(t)} \mathbf{x}_t$ using (5)
Update	$\mathbf{y}_t = \mathbf{U}^{(t-1)} \bar{\mathbf{y}}_t$
Update Statistics	$\mathbf{S}_i^{(t)}, i = 1 : 4$ using (14)
Compute	$\mathbf{Q}^{(t)}$ using (15)
Compute	$\mathbf{G}^{(t)}(p, q, \theta, \alpha)$ using (4)
Update	$\mathbf{U}^{(t)} = \mathbf{G}^{(t)}(p, q, \theta, \alpha) \mathbf{U}^{(t-1)}$
Update	$\mathbf{z}_t = \mathbf{G}^{(t)}(p, q, \theta, \alpha) \mathbf{y}_t$
Update	$\mathbf{W}^{(t)} = \mathbf{U}^{(t)} \mathbf{B}^{(t)}$ (only if needed ³)
end For	

Table 1. Adaptive GCMA Algorithm.

Once the statistics are updated, entries of $\mathbf{Q}^{(t)}$ are computed according to:

$$\begin{aligned}
q_{11}^{(t)} &= \frac{1}{4} (s_{1,pp}^{(t)} + s_{1,qq}^{(t)} - 2s_{1,pq}^{(t)}) \\
q_{12}^{(t)} &= \frac{1}{4} (s_{2,pq}^{(t)} + s_{2,pq}^{(t)} - s_{2,qp}^{(t)} - s_{2,qp}^{(t)}) \\
q_{13}^{(t)} &= \frac{1}{4j} (s_{2,pq}^{(t)} - s_{2,pq}^{(t)} - s_{2,qp}^{(t)} + s_{2,qp}^{(t)}) \\
q_{22}^{(t)} &= \frac{1}{4} (s_{3,pq}^{(t)} + s_{3,qp}^{(t)} + s_{1,pq}^{(t)}) \\
q_{23}^{(t)} &= \frac{1}{4j} (s_{3,pq}^{(t)} - s_{3,qp}^{(t)}) \\
q_{33}^{(t)} &= \frac{1}{4} (s_{3,pq}^{(t)} + s_{3,qp}^{(t)} - 2s_{1,pq}^{(t)})
\end{aligned} \tag{15}$$

Remarks:

- 1) Note that different strategies can be used for the selection of the rotation indices (p, q) in the adaptive scheme [12]. For simplicity, we have considered the automatic selection scheme where the indices p and q of the Givens rotations are automatically incremented at the current time instant, in such a way one sweeps, periodically, all matrix positions after each $d(d-1)/2$ time iterations.
- 2) Our adaptive algorithm costs $4md + O(d^2)$ flops per time instant. Comparatively, the adaptive ACMA [9] used below in our simulation comparison costs approximately $4md + O(d^3)$ flops per iteration.

5. SIMULATION RESULTS

To illustrate the performance of the proposed adaptive GCMA, we consider a 7×5 MIMO system (i.e. $d = 5, m = 7$). The inputs are i.i.d $8-PSK$ modulated sequences and the channel matrix \mathbf{A} is generated randomly at each Monte Carlo run but with controlled conditioning (its entries are generated as i.i.d. Gaussian variables). The separation quality is measured by the Signal to Interference and Noise Ratio (SINR) averaged over 500 Monte Carlo runs. The adaptation factor β is set to 0.99 in all the simulations.

In figure 1, we compare the convergence rates and separation quality of our algorithm with the adaptive ACMA while in figure 2, the plots represent the steady state SINR (obtained

after 1000 iterations) versus the SNR. One can observe, in this simulation context, that GCMA leads to high separation quality comparatively to the ACMA algorithm. The two algorithms have approximately the same convergence rate i.e. few hundreds of iterations are sufficient to reach the steady state level.

In figure 3, the SNR is set to $20dB$ and the plots represent again the steady state SINR versus the number of sources d . The number of antenna is set to $m = d + 2$. The two algorithms present the same performance in the case of two transmitted sources. When the number of sources increases, performance degradation is observed for the two algorithms with a significant advantage for the GCMA when the source number⁴ is less than 7.

In figure 4, the plots illustrate the algorithm performance when applied to non constant modulus signals. In this experiment the sources are generated from $16-QAM$ constellation. As we can see, the performance of the two algorithms are degraded comparatively to the constant modulus case presented in figure 1 and the GCMA still outperforms the adaptive ACMA.

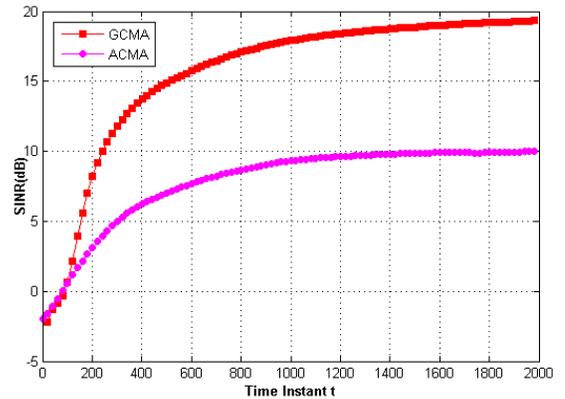


Fig. 1. SINR vs. Time Index.

6. CONCLUSION

A two steps adaptive constant modulus algorithm has been introduced using complex Givens rotation. The proposed algorithm is of moderate complexity and has the advantage of fast convergence rate and high separation quality. The simulation results illustrate its effectiveness with respect to the adaptive implementation of ACMA. They show, in particular, that the adaptive GCMA performs better than the adaptive ACMA when applied to more than two sources. The performance of the two algorithms are degraded when applied to the

⁴For large number of sources it will be better to use adaptive algorithm based on sliding window, e.g. [2], which are more expensive but more efficient in such a case.

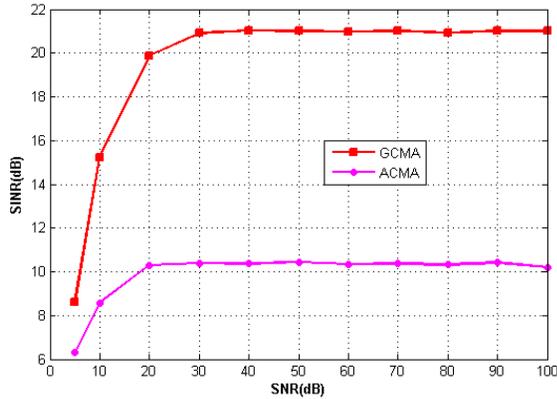


Fig. 2. SINR vs. SNR.

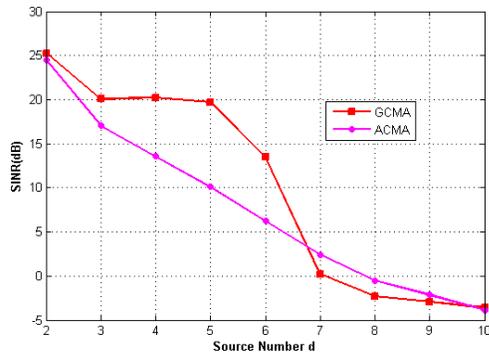


Fig. 3. SINR vs. Source Number.

separation of non constant modulus sources. To improve the performance of the proposed technique, more elaborated cost functions which combine the CMC with alphabet matching criteria have to be optimized, e.g. [10, 11].

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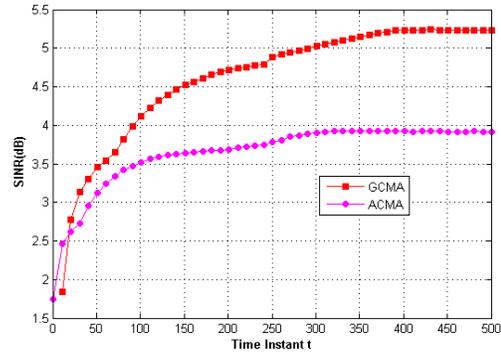


Fig. 4. SINR vs. Time Index, 16 – QAM sources.