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On the use of Dynamic Programming in eco-driving cycle computation for electric vehicles

D. Maamria [a], K. Gillet [a], G. Colin [a], Y. Chamaillard [a] and C. Nouillant [b]

***Abstract*— This paper considers the problem of eco-driving for electric cars. This problem is formulated as an Optimal Control Problem (OCP) aiming at minimizing the vehicle's energy consumption over fixed time and distance horizons. The impact of battery parameter variations and auxiliary power demands on the optimal vehicle velocity computation are studied from a model complexity viewpoint. Simulation results are presented and discussed to illustrate the suggested simplifications.**

I. Introduction

The expected depletion of fossil fuel sources, climate change due to pollution and an increase in overall energy demands are major challenges for the automotive industry. More generally, energy efficiency is increasingly becoming a major concern. For this purpose, highly efficient powertrain and lightweight automobiles are being developed. Furthermore, eco-driving is now considered as a major solution to reduce the energy consumption linked to transportation system.

In this context, several studies investigating the problem of the vehicle speed trajectory optimization have been reported [1]–[4]. In these studies, an Optimal Control Problem (OCP) to determine the velocity trajectory that minimizes the energy consumption under final time and distance constraints was formulated and solved. In particular, the eco-driving problem for electric vehicles was addressed in [1]–[3]. Usually, two state variables are considered in the OCP: the position and the speed of the vehicle.

In [1], the eco-driving problem was studied and solved for an electric car powered by a DC-type motor. The model used was based on an analytical expression of the electric power demanded by the electric machine. In [2], the same problem using similar modeling assumptions as [1] was addressed for an electric car powered by a permanent-magnet synchronous machine by taking into account physical limitations on the control actions. The advantage of the proposed solution is its relatively low computational time compared to Dynamic Programming (DP). Later, in [3], a more representative (realistic) model of the electric machine was used while neglecting the dependance of the internal battery resistance and the open circuit voltage on the battery State Of Charge (SOC) [5] as commonly assumed in the literature dealing with energy management system design for hybrid electric vehicles [6]–[9].

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In this paper, a similar eco-driving problem to [3] is considered. The dependance of battery parameters on the SOC is taken into account in the optimization. This increases the number of state variables from 2 (vehicle position and speed) to 3 (SOC, vehicle position and speed). The objective is to select the right level of modeling to optimize the accuracy/complexity trade-off. A main motivation is that the number of state variables greatly impacts the used numerical methods. Considering additional state variables increases the level of complexity and the computational burden. This observation holds for DP, and for methods using Pontryagin Minimum Principle (PMP) or direct formulations (e.g. collocation methods) [10], [11]. A special focus, in this paper, is on the quantification of the gain in energy consumption by including the SOC dynamics in the optimization.

The paper is organized as follows. In Section 2, the vehicle model is described. The calculation of eco-driving cycles and the questions addressed in this paper are detailed in Section 3. Section 4 discusses numerical and simulation results. In light of the results, some conclusions on the trade-off between the complexity of the models used to calculate eco-driving cycles and the optimality of the associated solutions are drawn.

II. Vehicle Modeling

A. Motion equations

The vehicle is modeled on the longitudinal axis. The motion of the vehicle is the result of the forces that are applied on its body. According to Newton's law of motion, the vehicle speed v satisfies the differential equation:

$$(m + m_{rot}) \cdot \frac{dv(t)}{dt} = F_t(t) - F_r(t), \tag{1}$$

where F_t is the traction force to be provided by the electric machine, F_r is the sum of resistance forces and m is the total vehicle mass. The term m_{rot} is an equivalent mass of the rotating parts. It accounts for the overall inertia of the wheels $(n_{tire} \cdot j_{tire})$ and for that of the electric machine (j_{rot}):

$$m_{rot} = \frac{n_{tire} \cdot j_{tire} + j_{rot}}{r_{tire}^2},$$

where r_{tire} is the wheel radius. The force F_r comprises the rolling resistance force, the aerodynamic drag force and a force due to the road grade. Its expression is given by:

$$F_r(t) = c_0 + c_1 \cdot v(t) + c_2 \cdot v(t)^2, \tag{2}$$

where $c_i,\ i = \{0, 1, 2\}$ are the coefficients of the road load equation (this expression of F_r was employed in [6], [12]).

This model considers only the forces in the longitudinal direction. Variations of friction parameters during curves, wind forces, and other disturbances are neglected.

B. Transmission model

The driver's torque demand and the vehicle speed are directly calculated from the wheel speed profile, elevation profiles and the transmission ratio. The resulting torque value T_{wh} can be positive (traction) or negative (braking). The electric machine torque T_e is related to the torque requested at the wheel T_{wh} by:

$$T_{wh}(t) = r_{tire} \cdot F_t(t) = \eta_t \cdot R_t \cdot T_e(t), \tag{3}$$

where η_t is the transmission efficiency and R_t is the constant motor-to-wheel transmission ratio. Similarly, the rotational speed ω_e of the electric machine is related to the vehicle speed v by:

$$\omega_e(t) = R_t \cdot \frac{v(t)}{r_{tire}}.$$

C. Electric machine model

The electric machine is modeled by a quasi-static map describing either the electric power or its efficiency. The electric power P_m consumed (in traction mode) or supplied to the battery (in recuperation mode) is of the form:

$$P_m = P_m(T_e, \omega_e),$$

where P_m is the electric power map of the electric machine.

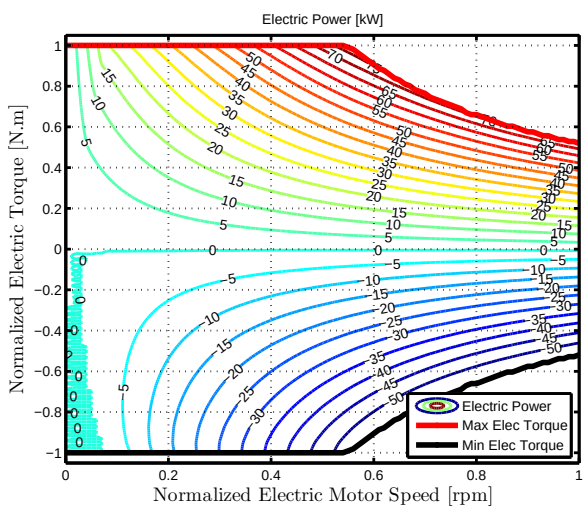

Fig. 1. Electric Machine Power [kW]

This map includes the losses in the electric machine and the power electronic devices. The electric machine torque is limited by speed-dependent upper and lower bounds of the form (bold red and black lines in Figure 1):

$$T_{emin}(\omega_e) \leq T_e \leq T_{emax}(\omega_e).$$

D. Auxiliary power demand model

A constant or a piecewise constant power demanded by auxiliaries P_{aux} (radio/tape player, lights, air conditioning and heating systems) is considered. This power is provided by the battery.

E. Battery model

The battery is usually represented by an equivalent circuit model comprising a voltage source U_{ocv} in series with an electric resistance R_b, both of which vary with ξ, the battery state of charge (SOC) [6], [13]. The expression of the battery current I_b is given by [6]:

$$I_b = \frac{1}{2R_b(\xi)} \left(U_{ocv}(\xi) - \sqrt{U_{ocv}^2(\xi) - 4R_b(\xi) \cdot P_b} \right),$$

where P_b is the power requested from the battery given by:

$$P_b = P_m + P_{aux}.$$

The dynamics of ξ is given by:

$$\frac{d\xi(t)}{dt} = -\frac{I_b(t)}{Q_0},$$

where Q_0 is the nominal battery capacity. In order to simplify the notation, the dynamics of ξ considering a given initial condition ξ_0 is written as:

$$\frac{d\xi(t)}{dt} = g(v(t), \xi(t), T_e(t)), \;\; \xi(0) = \xi_0.$$

The inner (electrochemical) battery power is defined by:

$$P_{ech}(v, \xi, T_e) = I_b(v, \xi, T_e) \cdot U_{ocv}(\xi).$$

The model parameters are summarized in Table I.

TABLE I

VEHICLE MODEL PARAMETERS

	Description	Value	Unit
m	Vehicle mass	1300	kg
r_{tire}	Wheel radius	0.34	m
n_{tire}	Wheel number	4	–
j_{tire}	Wheel inertia	4.28	kg·m^2
c_0	Constant coefficient of road load	120.86	N
c_1	Linear coefficient of road load	0.3	N/(m/s)
c_2	Quadratic coefficient of road load	0.0377	N/(m/s)^2
η_t	Transmission efficiency	0.925	–
R_t	Motor-to-wheel transmission ratio	4.7647	–
Q_0	Battery nominal capacity	28800	C
R_b	Internal battery resistance	$1.2 - 1.6$	$m\Omega$
U_{ocv}	Open circuit voltage	$3.45 - 4.10$	V

III. ECO-DRIVING

An eco-driving methodology consists in finding the optimal way to reduce the overall energy consumption [2], [14]. For a given road, the objective is to find the best speed profile minimizing the vehicle power consumption knowing that the vehicle starts from a point A at rest and must reach a destination point B in a duration t_f, with a zero velocity. This kind of question can be formulated as an OCP [4], [14].

A. Problem under consideration

The objective in this study is to investigate the impact of the battery parameters SOC dependance and the auxiliaries power demand on the optimal speed trajectory. The battery parameters SOC dependance is studied here from a model complexity viewpoint to find a trade-off between the computation time (induced by the model complexity) and

the optimality of the solution (energy consumption). Two questions are addressed:

1) What is the maximum benefit of considering the dependance of battery parameters in the optimal speed trajectory calculation?
2) What is the impact of the auxiliaries power demand P_{aux} on the optimal speed trajectory?

B. OCP formulation

The cost function (4) to be minimized is the electrochemical battery energy *in traction* over a fixed time window of duration t_f:

$$J(u) = \int_0^{t_f} P_{ech}(v(t), \xi(t), u(t))dt, \tag{4}$$

where the control variable u is the electric machine torque:

$$u(t) = T_e(t).$$

This optimization is carried out under the following differential equations:

$$\frac{dv(t)}{dt} = f(v(t), u(t)),\ v(0) = 0, \tag{5}$$
$$\frac{dx(t)}{dt} = v(t),\ x(0) = 0, \tag{6}$$
$$\frac{d\xi(t)}{dt} = g(v(t), \xi(t), u(t)),\ \xi(0) = \xi_0, \tag{7}$$

where x is the position of the vehicle and the function f is calculated by combining (1, 2, 3):

$$f(v, u) = \frac{1}{m + m_{rot}}(-c_0 - c_1 \cdot v - c_2 \cdot v^2 + \frac{\eta_t}{r_{tire}} \cdot R_t \cdot u).$$

Since the speed and the electric machine torque are limited and the final position and speed are set, the optimization must be performed under the following state and input constraints:

$$v(t) \in [0,\ v_{max}(x(t))], \tag{8}$$
$$u(t) \in [T_{emin}(\omega_e(t)),\ T_{emax}(\omega_e(t))], \tag{9}$$
$$x(t_f) = D, \tag{10}$$
$$v(t_f) = 0, \tag{11}$$

where D is the total traveled distance. The speed limits are given as a function of the vehicle position and not of time [15].

On the other hand, the final value of ξ is free as the traction of the vehicle is ensured only by the electric energy (no additional energy source is available on board as for hybrid electric cars).

To summarize, the following OCP can be defined:

$$(OCP): \quad \min_u J(u) \tag{12}$$

under the dynamics (5, 6, 7), state and input constraints (8, 9) and the final constraints (10, 11).

C. Simplified OCP

The dynamics of ξ is taken into account in the (OCP) described in (12) because of the internal battery resistance R_b and the open circuit voltage U_{ocv} dependance on ξ. To reduce the calculation time and the model complexity, *mean constant values* of R_b and U_{ocv} can be considered in the model used to calculate the optimal solution. In this case, the cost function (4) becomes of the form:

$$J_s(u) = \int_0^{t_f} P_{ech}(v(t), \bar{\xi}, u(t))dt,$$

where $\bar{\xi}$ is a fixed value used to calculate mean values of R_b and U_{ocv}. As the cost function J_s and the dynamics of v and x are independent of ξ, the number of the state variables is thus reduced from 3 (v, x, ξ) to 2 (v, x). The following simplified OCP can be defined:

$$(OCP_s): \quad \min_u J_s(u) \tag{13}$$

under the dynamics (5, 6), state and input constraints (8, 9) and the final constraints (10, 11).

D. Numerical solving method

The OCPs defined in (12) and (13) can be solved by numerous methods. The solution considered here is based on Dynamic Programming (DP) [16]. Considering the cost function J to be minimized:

$$J = \int_0^{t_f} L(X(t), u(t), t)dt,$$

from a mathematical viewpoint, Bellman's principle can be formulated as follows: let $t \in [0, t_f[$ and $\mathbf{X}(t) \in \mathbb{R}^n$ be given, then for all real $r \in [t, t_f]$, the cost-to-go function V satisfies:

$$V(X(t), t) = \min_{u \in \mathcal{U}} \left\{ \int_t^r L\big(X(\tau), u(\tau), \tau\big)d\tau + V\big(X(r), r\big) \right\}$$

This equation is solved recursively and backward. In order to reduce the calculation time, the method suggested in [3], [4], [17] is used, where an additional tunable term $\beta \cdot t_f$ is added to the cost function as a terminal cost:

$$\bar{J}(u) = \int_0^{t_f} P_{ech}(v(t), \xi(t), u(t))dt + \beta \cdot t_f,$$
$$\bar{J}_s(u) = \int_0^{t_f} P_{ech}(v(t), \bar{\xi}, u(t))dt + \beta \cdot t_f.$$

The constant tunable parameter β penalizes the final time to obtain almost the same time duration as the initial driving cycle. Thus, two new OCPs can be defined as follows:

$$(\overline{OCP}): \quad \min_u \bar{J}(u), \tag{14}$$

$$(\overline{OCP}_s): \quad \min_u \bar{J}_s(u) \tag{15}$$

To calculate the right value of the tunable parameter β, a root-finding method can be used to drive the final time error to zero as done in [15], [18]. It was shown in [3], [4] that the solutions of the problems $(\overline{OCP})$ and $(\overline{OCP}_s)$ converge to the solutions of (OCP) and (OCP_s) respectively when the obtained final time is almost the same as the initial driving cycle.

IV. Numerical Results

To compute an eco-driving cycle, the following constraints [14] extracted from an initial driving cycle have to be considered:

- the same final distance $x(t_f)$, the same number of stops and almost the same duration t_f as the initial driving cycle.
- the vehicle speed limits depending on the position of the vehicle (x).

To specify the speed limits, the following legal speed limits v_{lim} were applied:

$$v_{lim} = [30\ ,\ 50,\ 70,\ 90,\ 110,\ 130,\ 150]\ \text{(km/h)}.$$

The choice of this vector is not restrictive. Other speed limits can be used. The process of identifying the appropriate speed limit can be described in two steps:

1) For each time t, find the maximum value of j for which $v_{lim}(j) < v(x(t))$ and $v_{lim}(j) \geq v(x(t))$.
2) $v_{max}(x(t)) = v_{lim}(j)$.

Two normalized driving cycles are considered: the EUDC cycle with a duration of 360s and traveled distance of 6.9km and the Worldwide harmonized Light vehicles Test Cycle (WLTC) with a duration of 1574s and traveled distance of 22.7km.

The two OCPs defined in (14) and (15) are solved using DP with the following grid parameters: $\delta v = 0.01$m/s for the vehicle speed and of $\delta x = 20$m for the distance. For the control input u, a $\delta u = 1$N.m in the case of the EUDC Cycle and $\delta u = 2$N.m in the case of the WLTC cycle are used. The notation $\delta\xi$ refers to the step of ξ (SOC) used to solve the OCP (14). The initial value of the SOC is $\xi(0) = 90\%$.

To make a fair comparison between the solutions of problems (14) and (15), the control trajectory in each case is applied to a (forward) simulation model considering the variation of R_b and U_{ocv} as functions of ξ.

A. Impact of SOC dependent battery parameters

The internal resistance R_b and the open circuit voltage U_{ocv} depend on the SOC value. The maximum variation of R_b is about 12% while the variation of the U_{ocv} is 14% when the SOC changes between 90% and 20%. The impact of this variation on the optimal speed calculation is studied below for two driving cycles.

*1) **EUDC cycle**:* The speed trajectories obtained from the DP are shown in Figure 2 versus distance and in Figure 3 versus time. The electric energy consumption in [%] and the time α needed to run the DP are given in Table II.

TABLE II

Time α needed to run the DP and $\xi(0) - \xi(t_f)$ for EUDC cycle.

	$\xi(0) - \xi(t_f)$ [%]	α [s]
EUDC: Initial Cycle	18.74	–
Eco-EUDC: Simplified model	16.74	16
Eco-EUDC: $\delta\xi = 1\%$	16.73	764
Eco-EUDC: $\delta\xi = 0.5\%$	16.73	1537

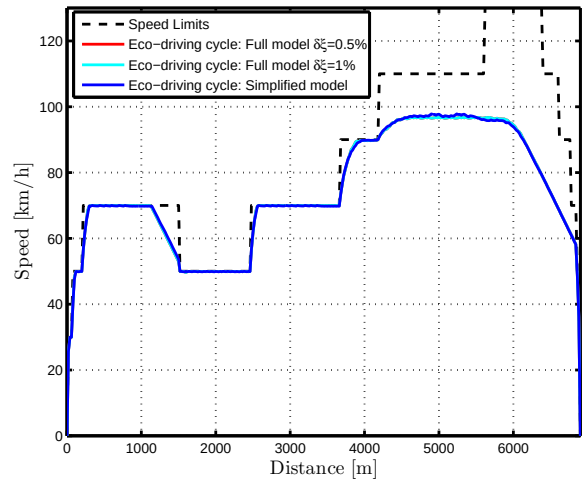

Fig. 2. Vehicle speed [km/h] vs distance [m] for EUDC cycle.

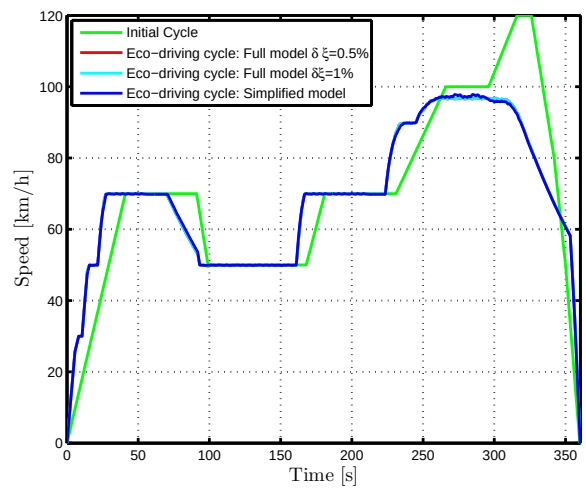

Fig. 3. Vehicle speed [km/h] vs Time [s] EUDC cycle.

Figures 2 and 3 show that the eco-speed trajectories satisfy the speed limits and that they are close. This can be confirmed from Table II where the difference in the final SOC between the cycles *Eco-EUDC* for $\delta\xi = 0.5\%$ and *Eco-EUDC* with the simplified model is negligible (0.01% on the energy consumption). On the other hand, the difference in the time needed to calculate these two cycles is significant (a ratio of 96).

*2) **WLTC cycle**:* For the previous case, the variation range of the SOC is not high. In order to ensure that the SOC sweeps a larger range, the WLTC cycle is used. The speed profiles obtained are shown in Figure 4 versus distance and in Figure 5 versus time. The electric energy consumption in [%] and the time α needed to run the DP are given in Table III. *we highlight that the stops are indicated in distance and not in time as for the speed limits.*

Figures 4 and 5 show that the eco-speed trajectories satisfy the speed limits. From Table III, the difference in the final SOC between the cycles Eco-WLTC for $\delta\xi = 1\%$ and

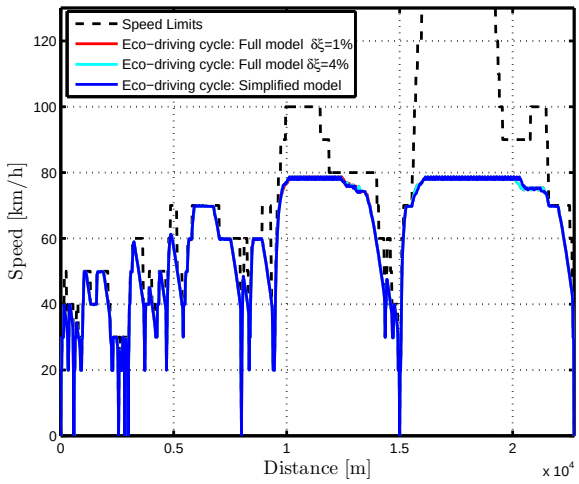

Fig. 4. Vehicle speed [km/h] vs distance [m] for WLTC.

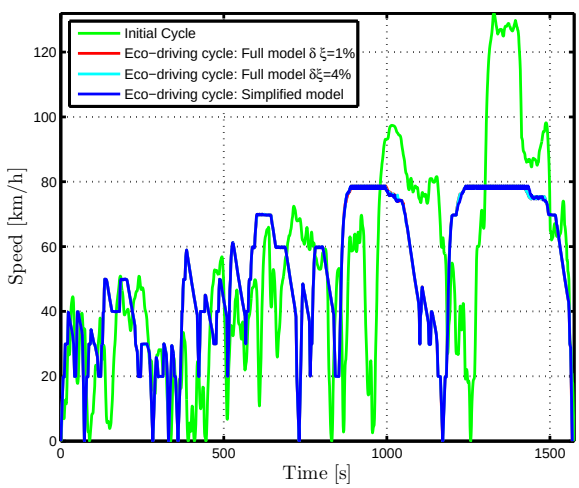

Fig. 5. Vehicle speed [km/h] vs Time [s] for WLTC.

Eco-WLTC with the simplified model is 0.01% (which is considered as negligible). On the other hand, the time needed to run the DP increases by a ratio of 116 when considering ξ in the OCP.

*3) **Conclusion**:* Because of the negligible sub-optimality induced and the reduction in the calculation time, the numerical results presented above suggest that it is sufficient to solve the OCP defined in (15) to calculate the optimal speed trajectory and to take the battery parameters dependance on the SOC into account only in the *(forward)* simulation model. *Note that similar analysis have been done for other normalized driving cycles: NEDC, Urban Artemis, Artemis Rural and Artemis highway cycles.*

B. Impact of the auxiliaries power demand

From the conclusion of the previous section, the OCP defined in (15) is considered in what follows. This optimization problem is solved in the case of the EUDC cycle (with a duration of 360s) for various values of β.

TABLE III

TIME α NEEDED TO RUN THE DP AND $\xi(0) - \xi(t_f)$ FOR WLTC.

	$\xi(0) - \xi(t_f)$ [%]	α [s]
WLTC: Initial Cycle	80.23	–
Eco-WLTC: Simplified model	62.82	30
Eco-WLTC: $\delta\xi = 4\%$	62.82	800
Eco-WLTC: $\delta\xi = 1\%$	62.81	3500

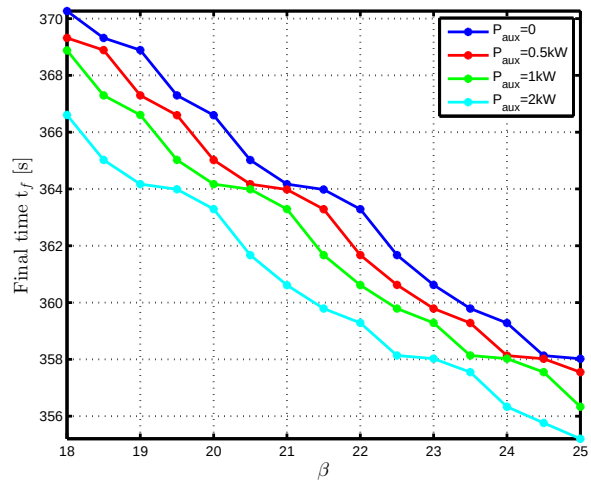

Fig. 6. Relation between β and the final time t_f[s].

*1) **Constant auxiliaries power demand**:* Various constant auxiliaries power demand P_{aux} are considered in the OCP (15). The final time t_f and the electric energy consumption are given in Figures 6 and 7. As one can see from Figures 6 and 7, the auxiliaries power demand P_{aux} shifts the final time t_f for the same value of β: the final time decreases when P_{aux} increases.

The objective here is to study the impact of P_{aux} on the speed trajectories for the same final time t_f. The speed trajectories having the same t_f for various values of P_{aux} are given in Figure 8. These results indicate that the optimal speed profiles are the same for the values of P_{aux} considered. The difference is only on the final SOC as illustrated in Figure 7 for a final time t_f around 360s.

*2) **Variable auxiliaries power demand**:* A variable auxiliaries power demand in [kW] defined by:

$$P_{aux} = \begin{cases} 1, \text{ if } x(t) \leq \dfrac{D}{3}, \\ 0.5, \text{ if } \dfrac{D}{3} < x(t) \leq \dfrac{2D}{3}, \\ 0, \text{ if } x(t) > \dfrac{2D}{3}, \end{cases} \tag{16}$$

is considered. The optimal speed trajectory obtained for this variable P_{aux} is compared to the optimal speed calculated for $P_{aux} = 0$ in Figure 9 (*the two solutions must have the same final time*). The optimal speed profiles are the same for the two cases.

*3) **Conclusion**:* The numerical results suggest that it is sufficient to set P_{aux} to zero in the model used to calculate

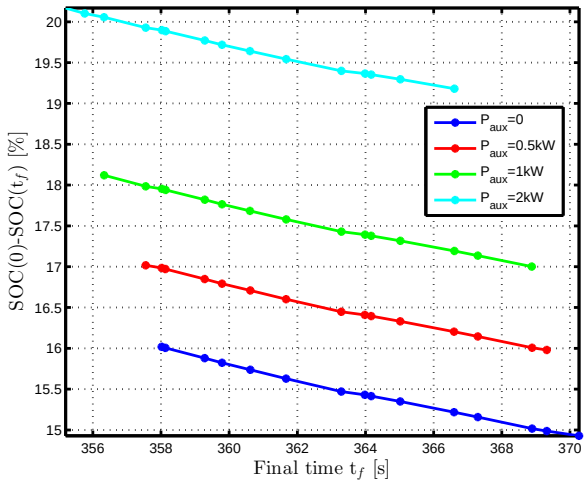

Fig. 7. Relation between t_f[s] and energy consumption $\xi(0) - \xi(t_f)$[%].

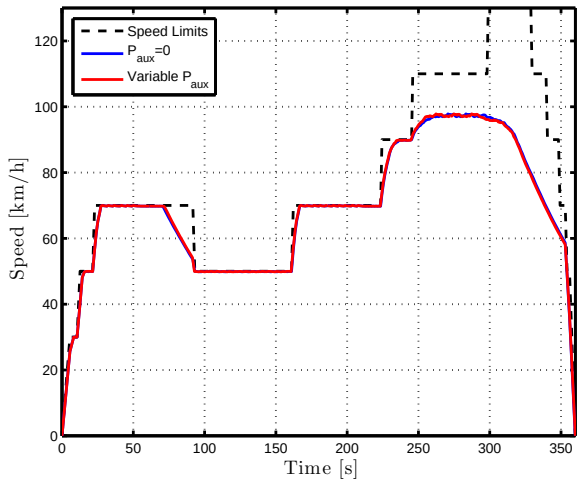

Fig. 9. Speed trajectories for a variable P_{aux} defined in (16).

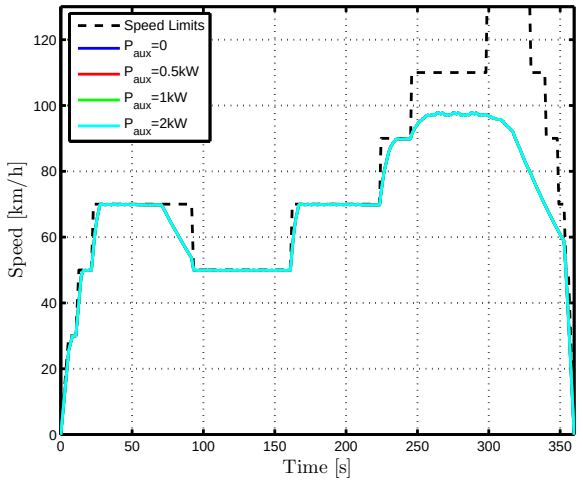

Fig. 8. Speed trajectories for various constant values of P_{aux}.

optimal speed trajectories and to take it into account only in the *forward* simulation model.

V. Conclusion

The eco-driving problem for electric vehicles has been addressed. This problem is formulated as an OCP aiming at minimizing the electric energy consumption. The studies conducted suggest that is not necessary to take into account the battery parameters dependence on the SOC and the auxiliaries power demand on the optimization problem. The impact of these simplifications on the optimal speed trajectory is negligible while ensuring a reasonable time to run dynamic programming.

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