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Energy Management of a Parallel Hybrid Electric Vehicle Equipped with a Voltage Booster

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Abstract: In this paper, the optimization problem of energy management for a parallel hybrid electric vehicle equipped with a Step-Up converter is resolved analytically using Pontryagin's Maximum Principle (PMP). The analytical method is based on convex models, which are identified from the reference models. A numerical method based on the reference models is also used in order to validate the analytical method by comparing their results. In this work, two optimization variables are considered: the power split between the Internal Combustion Engine (ICE) and the Electric Machine (EM) and the output voltage of the booster. The simulation results show that the analytical approach reduces considerably the computing time and has an very low suboptimality comparing to the numerical method.

Keywords: Energy Management Strategy, Hybrid Electric Vehicles (HEV), Step-Up Converter, Analytical Method, Convex optimization, Pontryagin's Maximum Principle (PMP).

1. INTRODUCTION

The continued increase of vehicles number in the world poses two major issues: energy and pollution. This is why, the automotive manufacturers design hybrid vehicles in order to reduce energy consumption and pollution emissions. The study presented in this paper focuses on the energy management of Hybrid Electric Vehicles (HEV). The energy management consists of calculating optimal controls that minimize the energy consumption. The studied vehicle is composed of an Internal Combustion Engine (ICE), one or more Electric Machines (EM), and a battery. This energy storage element delivers a continuous and almost constant voltage. Depending on the need, the adaptation of the electric energy between the battery and the EM is ensured by electric converters, among them the Step-Up which is a DC/DC converter. The role of the Step-up is to boost the battery voltage in order to increase the EM performance. The losses produced by the electric components depend strongly on the output Step-Up voltage. This is why we propose to control the voltage to minimize the electric consumption.

There are many approaches to design an optimal energy management strategy (Zhang et al., 2015), the most known are: deterministic Dynamic Programming (DP) (Pérez et al., 2006; Debert et al., 2010), stochastic DP (Johannesson et al., 2007; Asher et al., 2017), and Pontryagin's Maximum Principle (PMP) (Serrao et al., 2009; Kim et al., 2011; Stockar et al., 2011; Ahmadizadeh et al., 2017). While it can potentially give the globally optimal energy management, dynamic programming is computationally

expensive, which limits its application to low-order systems (typically two states). The PMP offers the possibility to compact the optimization problem by defining the Hamiltonian function to handle the balance between the fuel cost and other related constraints, typically the battery state of charge. However, the main difficulty of the PMP method remains in finding the co-state.

The PMP method is widely used in this area, both analytically and numerically. In Hadj-Said et al. (2017), we proposed an analytical method to optimize power split and gear box using PMP. In the latter, we used a linear electric model while in this paper the electric model is quadratic. In Elbert et al. (2014) and Ambühl et al. (2010), the optimal torque split and the engine state On/Off were computed analytically using the PMP approach for a serial hybrid electric bus. Pham et al. (2016) proposed to calculate in addition the optimal EM On/Off analytically using the PMP, while in Nüesch et al. (2014), the engine On/Off and gearshift strategies were given numerically by a combination of DP and PMP. In this study, the main contributions are: finding the optimal power split and the output Step-Up voltage analytically using PMP.

Concerning the voltage optimization, in Toshifumi Yamakawa (2011), the inventors proposed to implement a voltage setting map as a function of the rotational speed and the torque of the EM composed by two regions: a non-boost region and a boost region. These regions were determined by minimizing the EM losses.

The main objectives of this paper is to find an energy management solution that minimizes the energy consumed

by the vehicle. An analytical solution, based on PMP, is proposed for energy management of a parallel HEV.

This paper is organized as follows. In section 2, the reference and the analytical models are presented. In section 3, the off-line resolution of the optimization problem is proposed in two steps: first, the optimal power and then the voltage. In section 4, the implementation of the analytical solutions is presented. Finally, the simulation results obtained analytically in the MIL (Model In the Loop) are compared to the results obtained numerically in section 5. The purpose of this comparison is to validate the analytical solutions.

2. MODELING

As shown in Fig.1, the HEV consists of a battery, a Step-Up, an electric motor (EM), and an ICE delivering power to the wheels via a gearbox.

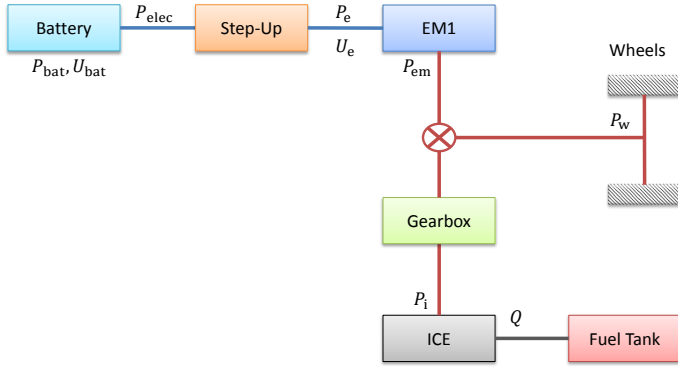


Fig. 1. HEV Powertrain

The wheel power P_w , demanded by the driver, is calculated from the vehicle speed set-point. So, the vehicle speed is considered as an input of the optimization, and is given by different cycles. The optimization variables are the output step-up voltage U_e and the mechanical power of the electric machine P_{em} .

2.1 Vehicle Model

The vehicle dynamics are governed by the following equations:

$$F_w(t) = m_{vec}\dot{v}(t) + F_{res}(t)[N] \quad (1)$$

$$T_w(t) = F_w(t) \cdot R_w[N.m] \quad (2)$$

$$\omega_w(t) = \frac{v(t)}{R_w}[rad/s] \quad (3)$$

$$P_w(t) = T_w(t)\omega_w(t)[W] \quad (4)$$

where F_w is the force at the wheels, $F_{res}(t) = F_{tires} + F_{aero}(t)$ the resistive force which includes the aerodynamic force ($F_{aero}(t) = 0.5 \cdot \rho \cdot S \cdot C_x \cdot v^2(t)$) and the tire resistance (F_{tires} , here assumed constant), $m_{vec}[kg]$ the total vehicle mass and $R_w[m]$ the wheel radius. P_w , T_w and ω_w are the power, the torque and the rotational speed of the wheels.

The relation between the rotational speed and those of the engine (ω_i) and the EM (ω_e) is given by:

$$\omega_i(t) = \omega_w(t)R_{Gear}(t) \quad (5)$$

$$\omega_e(t) = \omega_w(t)R_{EM} \quad (6)$$

where R_{EM} and R_{Gear} are respectively the electric ratio and the gear ratio which are determined in advance of optimization.

2.2 Reference Model

Engine The engine is modeled by its fuel consumption to deliver the mechanical power P_i . This consumption is expressed by the fuel flow (Q).

$$Q(t) = \dot{m}_{fuel}(T_i(t), \omega_i(t)) [g/s] \quad (7)$$

where $\dot{m}_{fuel}(T_i(t), \omega_i(t))$ is the fuel consumption map and T_i is the engine torque. The mechanical power generated by the engine is expressed by: $P_i = T_i\omega_i [W]$ and limited by two functions of ω_i :

$$\underline{P}_i(\omega_i(t)) \leq P_i(t) \leq \bar{P}_i(\omega_i(t)) \quad (8)$$

Electric Motor The electric motor model expresses the electric power produced by the EM which includes the mechanical power delivered and the losses obtained from the specific power loss of the EM. So, the electric power P_e has the following expression:

$$P_e(t) = P_{em}(t) + loss(T_e(t), \omega_e(t), U_e(t)) [W] \quad (9)$$

where $loss(T_e(t), \omega_e(t), U_e(t))$ is the electric motor losses map and T_e is the EM torque. The mechanical power generated by the EM is expressed by: $P_{em}(t) = T_e(t)\omega_e(t) [W]$ and limited by two functions of ω_e and U_e :

$$\underline{P}_e(\omega_e(t), U_e(t)) \leq P_{em}(t) \leq \bar{P}_e(\omega_e(t), U_e(t)) \quad (10)$$

Step-Up The Step-Up is modeled by its losses (P_S). They are expressed with respect to the electric power produced by the electric motors, the battery voltage, and the output voltage of the Step-up. In a step-up there are losses caused by conduction ($P_{Conduction}$) as well as losses caused by switching of diodes and IGBT ($P_{Switching}$). Therefore, P_S are given by (Badin, 2013):

$$P_S(i_{bat}, U_e) = P_{Conduction} + P_{Switching}[W] \quad (11)$$

$P_{Conduction}$ is given by:

$$P_{Conduction} = (R_{Coil} + rR_{IGBT} + (1-r)R_{Diode})i_{bat}^2 \quad (12)$$

where R_{Coil} is the coil resistance, R_{IGBT} is the IGBT resistance, R_{Diode} is the diode resistance and r is the duty cycle defined by:

$$r = \frac{U_e - U_{bat}}{U_e}$$

$P_{Switching}$ is given by:

$$P_{Switching} = V_0U_ei_{bat} + (rV_{IGBT} + (1-r)V_{Diode})i_{bat} \quad (13)$$

where V_{IGBT} is the IGBT voltage, and V_{Diode} is the diode voltage.

When $U_e = U_{bat}$, $P_{Switching} = 0$ and $P_{Conduction}$ is given by:

$$P_{Conduction} = \begin{cases} (R_{Coil} + R_{Diode})i_{bat}^2 & \text{if } P_{elec} \geq 0 \\ (R_{Coil} + R_{IGBT})i_{bat}^2 & \text{if } P_{elec} < 0 \end{cases}$$

P_S can take the following general form regarding i_{bat} :

$$P_S(i_{bat}, U_e) = U_S(U_e)i_{bat} + R_Si_{bat}^2 \quad (14)$$

where, $R_S = R_{Coil} + (1-r)R_{Diode} + rR_{IGBT}$

Battery The battery is modeled as a resistive circuit (Badin, 2013; Murgovski et al., 2012) and the battery power is given by:

$$P_{\text{bat}}(t) = OCV(SoC)i_{\text{bat}}(t)[W] \quad (15)$$

$$P_{\text{bat}}(t) = P_{\text{elec}}(t) + R_{\text{bat}}(SoC)i_{\text{bat}}^2(t) \quad (16)$$

$$P_{\text{elec}}(t) = P_e(t) + P_S(t) \quad (17)$$

By inserting (17) and (14) in (15), i_{bat} is calculated as follows:

$$i_{\text{bat}}(t) = \frac{U_{\text{tot}} - \sqrt{U_{\text{tot}}^2 - 4R_{\text{tot}}P_e}}{2R_{\text{tot}}} \quad (18)$$

where $U_{\text{tot}} = OCV(SoC) - U_S(U_e)$ and $R_{\text{tot}} = R_S + R_{\text{bat}}$.

The State of Charge (*SoC*) of the battery is defined as:

$$\dot{SoC}(t) = -\frac{i_{\text{bat}}(t)}{Q_{\text{max}}} \quad (19)$$

where $Q_{\text{max}}[C]$ is the maximum battery charge.

2.3 Convex Model

Some assumptions and approximations were made to make the models convex.

Engine The fuel flow Q is modeled by the "Willans Lines Model" as described in Rizzoni et al. (1999). Its analytical model is given by:

$$Q(P_i) = \begin{cases} a_1 P_i + Q_0 & \text{if } P_i \leq P_i \leq P_{\text{lim}} \\ a_2(P_i - P_{\text{lim}}) + Q_{\text{lim}} & \text{if } P_{\text{lim}} \leq P_i \leq \bar{P}_i \end{cases} \quad (20)$$

where Q_0 is the idle fuel consumption. The parameters Q_0 , P_{lim} and Q_{lim} depend on ω_i , while, a_1 , a_2 are assumed constant, where $a_1 \ll a_2$. Fig. 2 shows that the approximated engine model is sufficiently representative of the reference engine model. $RMSE_{\text{fuel}}$ is the Root Mean Square Error between the reference model (7) and the convex model (20).

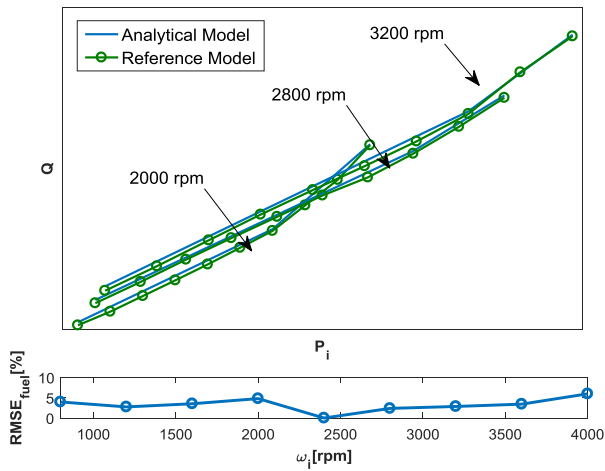


Fig. 2. Validation of the convex model of $Q[g.s^{-1}]$ in function of $P_i[W]$ (axes have been removed for confidentiality reasons)

EM, Battery, and Step-Up Concerning the electrical part, it is assumed that the open circuit voltage (*OCV*) is constant in the definition set of *SoE*. Therefore, the State of Energy (*SoE*) can be used instead of *SoC* as:

$$\dot{SoE}(t) = -\frac{OCVi_{\text{bat}}(t)}{E_{\text{max}}} = -\frac{P_{\text{bat}}(t)}{E_{\text{max}}} \quad (21)$$

where $E_{\text{max}} = OCVQ_{\text{max}}[J]$ is the maximal battery energy. The *SoE*[%] is limited by:

$$20 \leq SoE(t) \leq 80 \quad (22)$$

The analytical model of the battery power is given by:

$$P_{\text{bat}}(T_e, \omega_e, U_e) = \begin{cases} a^- P_{\text{em}}^2 + b^- P_{\text{em}} + c & \text{if } P_e \leq P_{\text{em}} \leq 0 \\ a^+ P_{\text{em}}^2 + b^+ P_{\text{em}} + c & \text{if } 0 \leq P_{\text{em}} \leq \bar{P}_e \end{cases} \quad (23)$$

where, a^\pm , b^\pm and c are modeled as following:

$$a^\pm(\omega_e, U_e) = \frac{\sum_{m=0}^2 \sum_{l=0}^2 \alpha_{ml}^\pm U_e^l \omega_e^m}{\omega_e^2} \quad (24)$$

$$b^\pm(\omega_e, U_e) = \frac{\sum_{m=0}^2 \sum_{l=0}^2 \beta_{ml}^\pm U_e^l \omega_e^m}{\omega_e} \quad (25)$$

$$c(\omega_e, U_e) = \sum_{m=0}^2 \sum_{l=0}^2 \gamma_{ml} U_e^l \omega_e^m \quad (26)$$

The P_{bat} model has been validated as shown in Fig. 3. Where, $RMSE_{\text{EM}}$ is the root mean square error between the reference model (15) and the convex model (23).

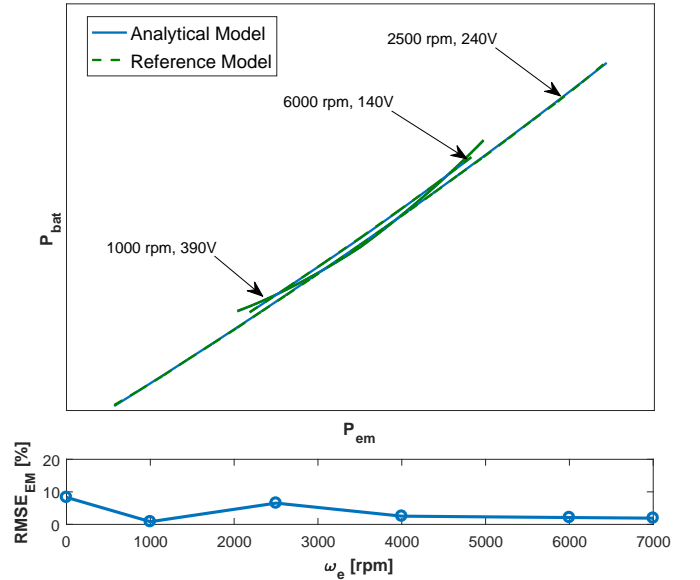


Fig. 3. Validation of the convex model of $P_{\text{bat}}[W]$ in function of $P_{\text{em}}[W]$ (axes have been removed for confidentiality reasons)

3. OFF-LINE OPTIMIZATION

In this section, the off-line optimization is presented. It is recalled that the rotational speeds ω_i and ω_e are determined from (5) and (6).

3.1 Power Split Optimization

Here, to optimize the EM power (P_{em}), we assume that the output Step-Up voltage U_e is known. In a parallel HEV, ICE and EM are mechanically connected to the wheels, therefore:

$$P_w(t) = P_{em}(t) + P_i(t) \quad (27)$$

The Power Optimization Problem (POP) to determine P_{em}^{opt} is formulated as:

$$POP : \begin{cases} \min_{P_{em}} J \\ \dot{S}oE(t) = -\frac{P_{bat}(P_{em}(t))}{E_{max}} \\ 0 \leq SoE(t) \leq 100 \\ P_i + P_{em} = P_w \\ \underline{P}_i(\omega_i, P_w) \leq P_i(t) \leq \bar{P}_i(\omega_i, P_w) \\ \underline{P}_e(\omega_e, U_e, P_w) \leq P_{em}(t) \leq \bar{P}_e(\omega_e, U_e, P_w) \end{cases} \quad (28)$$

where:

$$J = \int_{t_0}^{t_f} Q(P_i(t), \omega_i(t)) dt \quad (29)$$

SoE is the state, P_{em} is the control, and from (27) we obtain:

$$\begin{aligned} \underline{P}_i(\omega_i, P_w) &= \max(\underline{P}_i(\omega_i), P_w - \bar{P}_e(\omega_e, U_e)) \\ \bar{P}_i(\omega_i, P_w) &= \min(\bar{P}_i(\omega_i), P_w - \underline{P}_e(\omega_e, U_e)) \\ \underline{P}_e(\omega_e, U_e, P_w) &= \max(\underline{P}_e(\omega_e, U_e), P_w - \bar{P}_i(\omega_i)) \\ \bar{P}_e(\omega_e, U_e, P_w) &= \min(\bar{P}_e(\omega_e, U_e), P_w - \underline{P}_i(\omega_i)) \end{aligned}$$

To find the optimal power, the PMP is used. So, according to the PMP, minimizing J is equivalent to minimizing the Hamiltonian function which is calculated from (20) and (23), as follows:

$$H_{hyb}(P_i, P_{em}, P_w, \lambda) = Q(P_i) + \lambda(t)(P_{bat}(P_{em})) \quad (30)$$

where λ is the Langrange Factor.

The Hamiltonian function H_{hyb} is the sum of two piecewise functions. So, to find P_{em}^{opt} , first, we have to calculate the expression of H_{hyb} . This is done by considering the points where the functions Q and P_{bat} change their coefficients. These points are: $P_{em} = P_w - P_{lim}$ and $P_{em} = 0$. They are called here: "The Particular Points".

The general expression of H_{hyb} is:

$$H_{hyb}(P_{em}) = A_1(P_w - P_{em}) + A_0 + \lambda(B_2 P_{em}^2 + B_1 P_{em} + B_0)$$

A_1, A_0, B_2, B_1 and B_0 are determined, according to (20), (23) and (27), as follows:

- $P_{em} > P_w - P_{lim}$ then $A_0 = Q_0, A_1 = a_1$
- $P_{em} < P_w - P_{lim}$ then $A_0 = Q_{lim} - a_2 P_{lim}, A_1 = a_2$
- $P_{em} > 0$ then $B_1 = b^+, B_2 = a^+$
- $P_{em} < 0$ then $B_1 = b^-, B_2 = a^-$

Since H_{hyb} is convex, the optimum P_{em}^* is the solution of the following equation:

$$\frac{\partial H_{hyb}}{\partial P_{em}} = 0 \quad (31)$$

Then,

$$P_{em}^* = \frac{A_1 - \lambda B_1}{2\lambda B_2}$$

But H_{hyb} is a piecewise function then P_{em}^* should be compared to the particular points to determine A_1, B_1 and B_2 . In addition, P_{em}^* must be in the admissible set $[\underline{P}_e(P_w), \bar{P}_e(P_w)]$. Therefore, the following equations are studied regarding P_w and λ :

$$\begin{aligned} (a) \quad P_{em}^* = \bar{P}_e(P_w) &\Leftrightarrow \lambda = \lambda_1 = \frac{A_1}{B_1 + 2B_2 \bar{P}_e(P_w)} \\ (b) \quad P_{em}^* = (P_w - P_{lim}) &\Leftrightarrow \lambda = \lambda_2 = \frac{A_1}{B_1 + 2B_2(P_w - P_{lim})} \\ (c) \quad P_{em}^* = 0 &\Leftrightarrow \lambda = \lambda_0 = \frac{A_1}{B_1} \\ (d) \quad P_{em}^* = \underline{P}_e(P_w) &\Leftrightarrow \lambda = \lambda_3 = \frac{A_1}{B_1 + 2B_2 \underline{P}_e(P_w)} \end{aligned}$$

We can note that the particular factors $\lambda_0, \lambda_1, \lambda_2$ and λ_3 depend on P_w . Therefore, the optimal solution P_{em}^{opt} is calculated for all the admissible set: $P_w \in [\underline{P}_w, \bar{P}_w]$, where $\underline{P}_w = \underline{P}_i(\omega_i) + \underline{P}_e(\omega_e, U_e)$ and $\bar{P}_w = \bar{P}_i(\omega_i) + \bar{P}_e(\omega_e, U_e)$. For instance, if $P_w < \underline{P}_i \Rightarrow P_{em} < 0 \Rightarrow B_1 = b^-, B_2 = a^-$. And if $P_w - P_{lim} > \underline{P}_e$ then $\forall P_w \in [P_{lim} + \underline{P}_e, \underline{P}_i]$, P_{em}^{opt} is given as shown in the table of Fig. 4.

λ	0	λ_1	λ_{21}	λ_{22}	λ_3	$+\infty$
P_{em}^{opt}	\bar{P}_e	$\frac{a_1 - \lambda b^-}{2\lambda a^-}$	$P_w - P_{lim}$	$\frac{a_2 - \lambda b^-}{2\lambda a^-}$	\underline{P}_e	

Fig. 4. An example of P_{em}^{opt} according to P_w and λ

where, $\lambda_1 = \frac{a_1}{b^- + 2a^- \bar{P}_e}$, $\lambda_{21} = \frac{a_1}{b^- + 2a^- (P_w - P_{lim})}$, $\lambda_{22} = \frac{a_2}{b^- + 2a^- (P_w - P_{lim})}$, $\lambda_3 = \frac{a_2}{b^- + 2a^- \underline{P}_e}$.

3.2 Voltage Optimization

In this section, the optimization of the output Step-Up voltage will be presented. Here, we assume that the electric torque T_e is known. The PMP method is applied to solve the Voltage Optimization Problem (VOP):

$$VOP : \begin{cases} \min_{U_e} H_{hyb}(P_i, P_{em}, U_e, \lambda) \\ \dot{S}oE(t) = -\frac{P_{bat}(P_{em}, U_e)}{E_{max}} \\ 0 \leq SoE(t) \leq 100 \\ U_e \in [U_{bat}, \bar{U}_e] \end{cases} \quad (32)$$

where the Hamiltonian function is given as:

$$H_{hyb}(P_i, P_{em}, U_e, \lambda) = Q(P_i) + \lambda P_{bat}(P_{em}, U_e)$$

We can note that Q does not depend on U_e , therefore:

$$U_e^{opt} = \arg \min_{U_e} H_{hyb} = \arg \min_{U_e} P_{bat}$$

So, to find U_e^{opt} , we must solve the following equation:

$$\frac{\partial P_{bat}}{\partial U_e} = 0$$

According to (23), P_{bat} can be written in the following form:

$$P_{bat}(T_e, \omega_e, U_e) = U_2(T_e, \omega_e) U_e^2 + U_1(T_e, \omega_e) U_e + U_0(T_e, \omega_e) \quad (33)$$

Then, $\frac{\partial P_{bat}}{\partial U_e} = 0 \Leftrightarrow U_e^*(T_e, \omega_e) = \frac{-U_1(T_e, \omega_e)}{2U_2(T_e, \omega_e)}$

where:

$$U_2(T_e, \omega_e) = \begin{cases} \sum_{m=0}^2 (\alpha_{m2}^- T_e^2 + \beta_{m2}^- T_e + \gamma_{m2}) \omega_e^m & \text{if } T_e \leq 0 \\ \sum_{m=0}^2 (\alpha_{m2}^+ T_e^2 + \beta_{m2}^+ T_e + \gamma_{m2}) \omega_e^m & \text{if } T_e \geq 0 \end{cases}$$

$$U_1(T_e, \omega_e) = \begin{cases} \sum_{m=0}^2 (\alpha_{m1}^- T_e^2 + \beta_{m1}^- T_e + \gamma_{m1}) \omega_e^m & \text{if } T_e \leq 0 \\ \sum_{m=0}^2 (\alpha_{m1}^+ T_e^2 + \beta_{m1}^+ T_e + \gamma_{m1}) \omega_e^m & \text{if } T_e \geq 0 \end{cases}$$

Finally, since $U_2(T_e, \omega_e) > 0 \forall T_e, \omega_e$, U_e^{opt} is given as a function of T_e and ω_e (Fig. 5), by:

$$U_e^{\text{opt}} = \begin{cases} U_{\text{bat}} & \text{if } U_e^*(T_e, \omega_e) \leq U_{\text{bat}} \\ U_e^*(T_e, \omega_e) & \text{if } U_{\text{bat}} \leq U_e^*(T_e, \omega_e) \leq \bar{U}_e \\ \bar{U}_e & \text{if } U_e^*(T_e, \omega_e) \geq \bar{U}_e \end{cases} \quad (34)$$

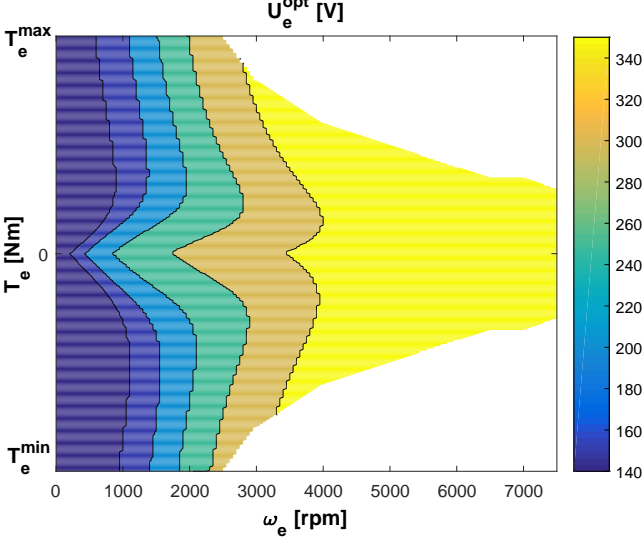


Fig. 5. The optimal Voltage regarding T_e and ω_e (the torque axe has been removed for confidentiality reasons)

4. ON-LINE OPTIMIZATION

This section describes how the analytical and numerical approaches are implemented in the Model In Loop (MIL) structure.

Fig. 6 shows the on-line optimization process in the MIL. Actually, by taking into account the engine, the EM and the Step-Up dynamics, at an instant t , the power optimization receives the current voltage U_e^c , while the voltage optimization takes the current electric torque T_e^c .

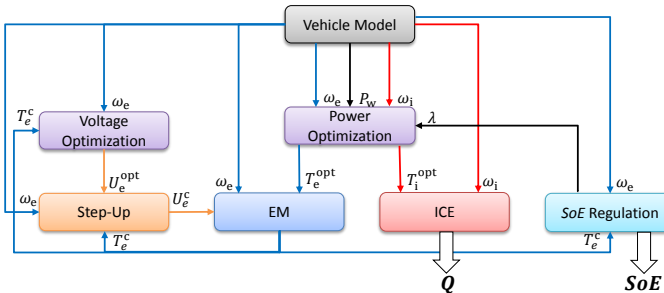


Fig. 6. On-Line Optimization for a Parallel HEV

For the optimal power, as shown in Fig.4, $P_{\text{em}}^{\text{opt}}$ is implemented in the form of tables regarding P_w and λ . Then, at an instant t , the expression of $P_{\text{em}}^{\text{opt}}$ is found by placing $P_w(t)$ and $\lambda(t)$ with respect to their intervals. Concerning the voltage U_e , the optimal solution given in (34) is implemented.

The co-state λ is determined by a PID regulator of the SoE.

The diagram in Fig.7 explains the numerical approach to resolve POP and VOP by applying PMP and using the reference models. First, at every instant t , the controls P_{em} and U_e are meshed from their minimum to their maximum. Then, the numerical value of H_{hyb} is calculated using the complete models of Q and P_{bat} (given in (20), (23), (16) and (11)). Finally, the optimal control $P_{\text{em}}^{\text{opt}}, U_e^{\text{opt}}$ is the one which corresponds to the minimum value of H_{hyb} . In this approach, the optimality of U_e^{opt} and $P_{\text{em}}^{\text{opt}}$ depends on the meshing. This is why the numerical method was tested in simulation for a meshing of 10 points (called "Num₁" in section 5) and 100 points (called "Num₂" in section 5).

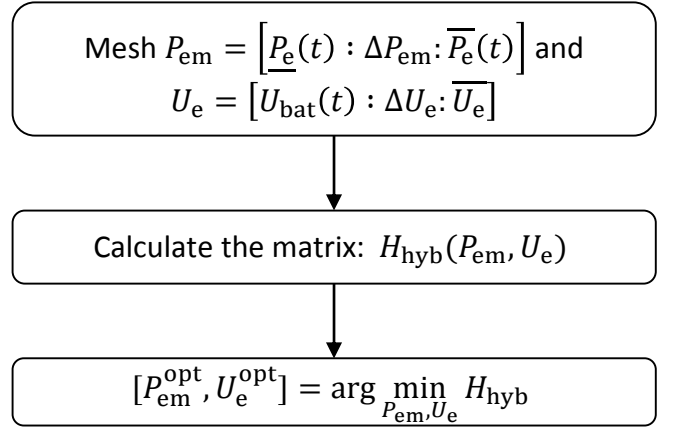


Fig. 7. Implementation of the numerical method

5. SIMULATION RESULTS

In this section, the fuel consumption and SoE trajectory results of the analytical method are compared to those of the numerical method in order to establish the performances of the analytical method. The fuel consumption (FC) and the electric losses (EL) are obtained by applying the strategies on the reference models.

Table 1 shows that the analytical method has almost the same fuel and electric consumptions as the ones found by the numerical methods for all studied cycles. For the urban driving, the analytical method is better than "Num₁". In addition, the average values of the Computing Time (CT) and Memory Space (MS) of the analytical method are lower than those of the numerical method.

Fig. 8 shows that both methods provide a similar SoE trajectory and almost the same optimal controls for the ARTEMIS highway cycles.

6. CONCLUSIONS

In this paper, an analytical approach has been presented and applied to calculate the energy management strategy for a parallel HEV. The results of the comparison show that the analytical method, which is based on analytical models, provides an optimal solution close to the one given by the numerical method, thereby validating the

Cycle	Strategy	FC	EL	CT	MS
		[L/100km]	[Wh]	[ms]	[Bytes]
ARTEMIS Highway	Analytical	6.26	616	0.5	44
	Num ₁	6.26	602	1.2	80
	Num ₂	6.25	615	2	530
ARTEMIS Urban	Analytical	5.28	195	0.5	44
	Num ₁	5.30	192	1.2	80
	Num ₂	5.24	193	2	530

Table 1. Fuel Consumption, Electric Losses, Computing Time and Memory results

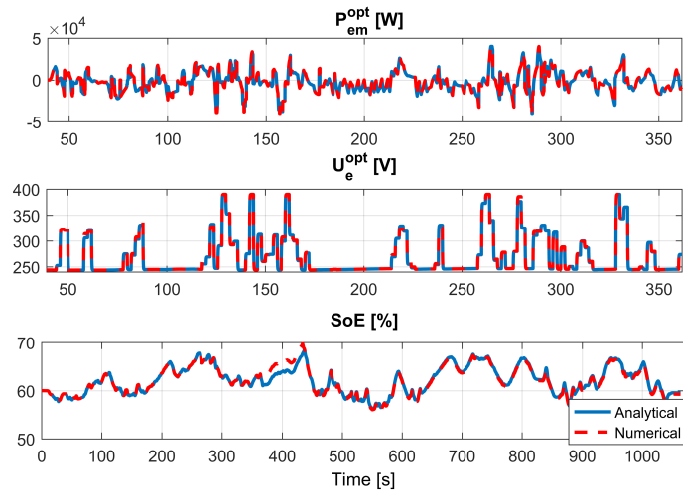


Fig. 8. The optimal controls and the SoE trajectory obtained by the numerical method "Num₂" and the analytical method for the ARTEMIS Highway cycle

approximated generic models. The implementation of the analytical solutions is easier and requires less computing time than the numerical resolution. This encourages their use for embedded optimal control.

As perspectives, the analytical method will be applied to other HEV architectures (serial, serial-parallel), and to more complex configurations (several EM and batteries). This strategy will be implemented for real-time energy management. The robustness analysis will be studied with respect to the parameters of the EM model and those of the engine.

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