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**Okun's law in the US: new insights in time and frequency**

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## Okun's law in the US: new insights in time and frequency

### Abstract

This paper investigates the direction of co-movement between cyclical components of unemployment and output in the US by using the wavelet methodology over the period 1948Q1-2020Q4.

The results show that the direction of co-movement is rather idiosyncratic in the short-term with Okun's effect being validated only during economic turbulences. The evidences also reveal that the direction of interaction is related to the economic shocks and their internal or external origins. The findings related to pandemic disease should be interpreted with caution. In the medium-term, Okun's law follows the demand approach, as the Keynesian unemployment is evidenced at the business cycle with the GDP negatively running unemployment. Noteworthy here is that the link is sensitive to economic volatility, disappearing during low volatility level. In the long-term, the results show that the supply approach is operative. In this case, unemployment negatively drives GDP, proving classical unemployment.

**Key words:** Unemployment gap; Output gap; Okun's law; Wavelet; US

**JEL classification:** C14, E24, O40

### Introduction

Starting with Okun's (1962) paper, many researchers have deeply investigated the negative relationship between the unemployment gap and the output gap. Over the years, the Okun's (1962) effect in the case of the United State (US) has become a huge topic of debate.

In his initial work, Okun (1962) connects cyclical output with cyclical unemployment by showing that a one percentage point rise in the unemployment rate determines an output reduction of 3.2% with respect to its potential level. A large body of literature is devoted to combat the value of the coefficient of Okun's law as well as the entire procedure. Moosa (1999) shows that the source of biases comes from the type of link (i.e., contemporaneous versus lagged), lack of control variables, difference between demand and supply shocks, use of cyclical components by following short- and long-run effects, selected method of estimation and considered sample.

Unfortunately, no study seriously treats the co-movement direction issue in an extended way, although this has crucial policy implications. More precisely, there is no contribution that clearly explains when it is opportune to follow the demand approach (i.e., output as the explanatory variable) or the supply approach (i.e., unemployment as the explanatory variable). The demand approach is the ground of Keynesian unemployment, being registered at the business cycle. Herein, the lack of effective demand for goods and services compresses the production because fewer workers are needed. More precisely, the fall of output generates unemployment. The supply approach is related to the classical unemployment model, with its root in the level of real wages. In this case, when the real wages are set above the market-clearing, the number of job-seekers exceeds the number of vacancies. For higher real wages, the producers cannot employ all available workers and some of them remain unemployed. The rising unemployment leads to a fall in employment that causes a reduction of output. Herein, the unemployment runs output.

The internal and external shocks are also neglected in Okun's law, and we often observe the implications of production costs or supply chain processes. Collins and Park (1989) suggest that the internal shocks are unexpected or unpredictable events that affect an economy coming from its internal system (e.g., economic disequilibrium, politico-institutional changes, civil wars, etc.), while the external shocks are turbulences with origins outside the economic system (e.g., sudden rise of international oil prices or interest rates, trade embargoes, widespread adoption of technologies, pandemic diseases, etc.). Moreover, Ferreira et al. (2010, p. 303) show that "as opposed to external shocks that affect various countries in a systematic fashion, the breaks associated with internal dynamics should not present strong regularities across countries."

The main aim of paper is not to revisit the procedure of the Okun's law relationship of coefficient calculus, but rather to clarify the direction of co-movement between cyclical components of unemployment and output using the wavelet methodology. The covered period is 1948Q1-2020Q4. The wavelet methodology is superior to the classical time-domain methodology, offering deep details about the intensity of co-movement between variables over time and frequencies by following their lead-lag status on short-, medium- and long-runs (Mutascu and Sokic, 2020). Such technique also facilitates the checking for nonlinearity between variables, excluding any kind of classical time-domain approach (i.e., dataset splitting, polynomial test, use of dummy variables, threshold tests, etc.). By following historical US economic events, the method also allows to 'extract' deep information at the level of sub-periods of time. It indicates whether the origins of unemployment come from demand side (i.e., unemployment is the result of declining aggregate demand reflected by output) or supply side (i.e., unemployment is driven by the level of wages affecting the final output). Not at least, the turbulences are historically localized either through their cost implications (e.g., oil crises) or via different shocks coming from inside the economy (e.g., economic crises, terrorist attacks, civil wars, political system changes).

The theoretical ground of the study is Okun's law that describes a negative relationship between the unemployment gap and the output gap. The outputs are rather idiosyncratic in the short-term, with evidence connecting the origins of economic shocks to the direction of co-movement. The results show different directions of co-movement with various time durations, as Okun's law effect covers medium- or long-term horizons. The time-horizons are purely conventional, being set as follows: up to 1 year: short-term, 1-10 years: medium-term, and more than 10 years: long-term. They are inspired from the average of business-cycle duration (2-10 years) in the US after 1948, as reported by NBER (2021). This period shapes the medium-term business cycle concept, introduced by Comin and Gertler (2006). Short-term captures the periods characterized by turbulences in which the economy does not have the capacity to equilibrate, which is in contrast to the long-term horizon in that the economy exhibits large business cycles, with self-correction mechanisms.

The contribution of this paper to the literature is threefold. First, along with Aguiar-Conraria et al.'s (2020) contribution, to the best of our knowledge, our study represents a pioneering work on the Okun's law topic that follows the wavelet technique. Aguiar-Conraria et al. (2020) mainly focus on the magnitude of Okun's coefficient, validating the effect on both standard and medium business-cycle frequencies. As they did not link the results with the types of unemployment and their related policy implications, our work tackles exactly those issues. More precisely, we highlight the existence of both Keynesian and classical unemployment as well as the sensitivity of the effect to external and internal shocks. Such information is essential to design any policy corrections.

Second, unlike Aguiar-Conraria et al. (2020), in our paper, productivity is called upon to explain how Okun's effect evolves over both time and frequency under periods with different economic volatility. The approach is inspired from Benigno et al. (2015), who claim that the economic volatility influences the probability of adverse shocks which modify the level of wages affecting unemployment. The economic volatility is observed via the wavelet tool, both unemployment and output historically exhibiting periods with high and low volatility.

Third, in comparison to Aguiar-Conraria et al. (2020) who cover the period 1949Q1-2018Q2, our analysis considers an extended updated sample (i.e., 1948Q1-2020Q4) as well as a dataset with monthly frequency (i.e., 1948M1-2020M12). This allows us to see how the co-movement between unemployment and output evolves inside the month, checking also for robustness. In parallel, we use an alternative wavelet method (i.e., wavelet cohesion) to additionally reinforce the results. Compared to the classical wavelet tool (i.e., wavelet coherency), this allows us to better 'filter' both sign and intensity of co-movement between unemployment and output across the frequencies.

The rest of the paper is organized as follows: Section 2 presents the literature, Section 3 describes the data and methodology, Section 4 shows the empirical results, and Section 5 concludes.

## Literature

The literature devoted to Okun's law is well extended, offering heterogeneous results. In this respect, Durech et al. (2014) systematize the main contributions and find four strands of literature.

The **first strand** is related to the level of Okun's coefficient and is exclusively focused on the US case. For example, Prachowny (1993) introduces into the equation the weekly hours and capacity utilization. He claims that the connection going from cyclical unemployment to output gap is around -0.67 lower in absolute value than Okun's indicated level of -3.2. Weber (1995) and Moosa (1999) show that Okun's coefficient is around 3 when using quarterly data. Different from that, Attfield and Silverstone (1997) and Coen and Hickman (2006) arrive at coefficients of -2.25 and -1.90, respectively. Attfield and Silverstone (1997) use a cointegration approach, while Coen and Hickman (2006) follow classical regressions and the Hodrick-Prescott filter (HP filter) to subtract the cyclical components of the series. Galí et al. (2012) propose a new theoretical perspective by incorporating the new Keynesian model. They ultimately consider that such an approach allows to correctly estimate the measure of the output gap. In the same note but using a dynamic stochastic general equilibrium (DSGE) model with financial frictions, Gelfer (2020) reports more nuanced findings. He states that investment and finance turbulences have a larger impact of employment and unemployment compared to productivity and other supply-side shocks.

Unlike them, Ho (2014) introduces economic volatility into equation, claiming that the magnitude of Okun's coefficient fell during the Great Moderation (1984-2007). More precisely, the coefficient reduced from -1.87 to -1.37 after the year 1984, perfectly coinciding with the period of Great Moderation defined by Dornbusch et al. (2013) as being the period from 1984 to 2007, characterized by low economic volatility. Finally, it is noteworthy to mention the contribution of Grant (2018), who dedicates his work to estimating the time-varying parameter of Okun's law based on the potential output and natural rate of unemployment. Grant (2018) finds

that the widely accepted Okun's coefficient of -2 has low probability to be validated during the Great Recession.

The **second strand** of literature offers comparisons between countries regarding the magnitude of Okun's coefficient, the main results revealing a huge heterogeneity. For example, Ball et al. (2017) focus on the case of the US since 1948 in parallel with a dataset, and include 20 other advanced economies since 1980. They find that Okun's effect is stable over time, being strong in almost all countries. Moreover, the effect does not reveal a significant change during the Great Recession.

Guisinger et al. (2018, p. 239) highlight that those differences across countries come from "employment protection and minimum wage laws, the power of trade unions, and demographics." In this light, the authors indicate an extended set of papers, from Paldam (1987), Kaufman (1988) and Moosa (1997), to most recent contributions of Cazes et al. (2013) and Hutengs and Stadtmann (2013).

The **third strand** covers the non-linear approaches, with an accent on Okun's coefficient magnitude across the phases of the business cycle as well as time-variation under different structural breaks. One of the first such contributions belongs to Moosa (1999), who finds that the coefficient differs on the short-run compared to the long-run. The business cycle seems to have a strong importance, as the results differ during upturns and downturns, as Harris and Silverstone (2001) and Cuaresma (2003) point-out. Other authors, such as Virén (2001) and Fouquau (2008), prove the influence of unemployment level. The threshold models open a new wave in Okun's law analysis. For example, Knotek (2007) and Beaton (2010) use the time-period threshold. Knotek (2007) finds that Okun's law is not stable over time in the US. Beaton (2010) compares the US with Canada and similarly shows that there is a structural instability in both countries, with the unemployment rate being very sensitive under movements in output growth.

The characteristics of the labor market are also successfully used. Malley and Molana (2008) consider the labor implication, while Huang and Lin (2008) introduce the productivity effect. Mayes and Virén (2009) perform interesting threshold analyses focusing on the Economic and Monetary Union (EMU) status, and Neely (2010) refers to market regulations.

Focusing on the US, Aguiar-Conraria et al. (2020) have one of the first studies to investigate Okun's law via the wavelet tool. The span covers 1948:Q1-2018Q2. Their main findings are that Okun's coefficient continuously increased since the 1960s, with the exception of the 1985–1995 period. Additionally, the authors claim a strong Okun's effect at the business cycle level as well as at low frequency.

The **last strand** of literature includes papers devoted to the regional level, using the pool models as the main empirical tools. The outputs are very different depending on the time-period, country and methods followed. One of the first contributions in the field belongs to Freeman (2000), who investigates the infra-country level in the US. He stresses that Okun's coefficient is stable across the regions of the US at the level of 2, over all analyzed periods. Pereira (2014) underlines the importance of regional spillovers in defining the relationship at the national level and highlights strong asymmetries in Okun's law across the business cycle. Many other papers investigate the European space, with various outputs: Kangasharju and Pehkonen (2001) - Finland; Apergis and Reztis (2003) - Greece; Binet and Facchini (2013) - France; Porras-Arena and Martín-Román (2018) - Spain; and Durech et al. (2014) - Czech Republic and Slovakia.

Overall, Okun's law describes a relationship between the cyclical component of two variables. The literature illustrates that both **directions of co-movement** were explored, running from unemployment to output and vice-versa. Since Okun's (1962) contribution, the study of direction

was quasi-completely neglected although it is crucial to formulate adequate decisions to combat unemployment and stimulate economic growth. The direction running from unemployment to output is related to the supply approach, where real wages play an important role. Herein, no policy corrections are needed as the economy has a self-corrective mechanism. Otherwise, a direction driving from the output to unemployment suggests a demand approach, the output being strongly interlinked with the aggregate demand. In this case, the control of unemployment and output should be done by conducting macroeconomic policies focused on aggregate demand and growth.

Barreto and Howland's (1994) study is one of the first attempts dedicated to this issue, their conclusion using Blackley's (1991) contribution. Citing Blackley (1991, p. 641), they state that the regressions of output on unemployment is more appropriate when Okun's law is used "(1) as a parameter in formulas which calculate potential GNP, ... or (2) as a way to measure the opportunity cost in foregone output of unemployment when it exceeds its natural, full employment rate."

Otherwise, the reverse co-movement direction, driving from unemployment to output, is opportune when Okun's law is used "(3) in econometric models to forecast unemployment in response to anticipated changes in real GNP," citing the same (Blackley, 1991, p. 641). Guisinger et al. (2018, p. 240) also support the selected direction of co-movement by underlining that "as it is common to assume that other shocks affect output more than unemployment, we prefer to treat output deviations as the dependent variable."

In this literature context, two main gaps can be identified. The first gap relates to the missing papers discriminating between Keynesian and classical unemployment in Okun's effect and their importance in drawing up adequate corrective policies. The internal and external shocks are also neglected. Although Aguiar-Conraria et al. (2020) explore Okun's law by using the wavelet methodology, they focus on coefficient magnitude and totally ignore the economic 'flavor' of the aforementioned approaches. The second gap is given by the fact that just a few papers consider the implications of productivity on the stability of Okun's effect (Huang and Lin, 2008). The productivity plays an important role on both unemployment and output, especially during the periods with high economic volatility (Benigno et al., 2015).

This paper addresses both gaps by thoroughly exploring the co-movement between cyclical components of unemployment and output in the case of the US with the wavelet method over the period 1948Q1-2020Q4.

## Data and methodology

### Data

We consider two time-series in order to investigate Okun's law in the US by using the wavelet approach: unemployment rate and GDP. The sample covers the period 1948Q1-2020Q4, having quarterly frequency. Both variables are seasonally adjusted.

The **unemployment rate** captures the number of unemployed people as a percentage of the labor force. The labor force refers to "people 16 years of age and older, who currently reside in 1 of the 50 states or the District of Columbia, who do not reside in institutions (e.g., penal and mental facilities, homes for the aged), and who are not on active duty in the Armed Forces" (FRED, 2021). As the unemployment rate series is available with monthly frequency only, the quarterly version is constructed by using the quarter average of related monthly values.

The **Gross Domestic Product (GDP)** represents the real gross domestic product, in billions of chained 2012 dollars.

Two additional variables are used to support the main findings: participation rate and productivity index.

Participation rate measures the number of employed and unemployed persons as a percentage of total working-age population. This variable allows us to connect the GDP with participation rate in order to check the sensitivity of Okun's law to different kinds of jobless measures. It is noteworthy that the unemployment rate captures only the officially registered unemployed persons, not capturing the rest of jobless persons.

The productivity index relates the business output in terms of expenditure with the income, the productivity being captured in an economic sense. More precisely, it shows how different factors of production (i.e., inputs) affect the overall output (Sickles and Zelenyuk, 2019). The variable links the GDP with productivity, this nexus having complex implications during the periods with different economic volatilities (Benigno et al., 2015).

For unemployment rate, GDP and participation rate, the source of data is Federal Reserve Economic (FRED) Data, Federal Reserve Bank of St. Louis, online database (2021). The productivity index is taken from US Bureau of Labor Statistics (BLS), online database (2021). Following Okun's (1962) approach, the HP filter (Hodrick and Prescott, 1997) is used to extract the cyclical components of the series, ground of estimations.

The variables are treated as percentages in the case of unemployment rate, productivity index and participation rate, while natural GDP appears in its natural logarithm form. The variables are considered in their level, as the time-frequency analysis does not require stationary property of the series. In this context, Aguiar-Conraria et al. (2008, p. 2877) show that the wavelet tool is used “to quantify the degree of linear relation between two non-stationary time series in the time-frequency domain.”

## Methodology

We implement the wavelet approach to analyze Okun's law in the case of the US over the period 1948Q1-2020Q4. Although it is widely considered in digital image processing, medicine, geophysics, acoustics or astronomy, the method has been successfully adapted to economic investigations. Unlike the classical time-domain tools, the wavelet compactly shows the sign, direction and duration of co-movement between variables at different frequencies over time.

The wavelet shows that “true economic relationship among variables can be expected to hold at disaggregated (scale) level rather than at the usual aggregation level,” as Dar et al. (2014, p.3) note. Such approach allows us to observe not only the intensity and direction of interaction between the cyclical components of unemployment and GDP, but also how that link varies across different frequencies over time.

Generally, Okun's law supposes that:

$$y_t - \bar{y}_t = \alpha_0 + \alpha_1 (u_t - \bar{u}_t) + \varepsilon_t, \quad (1)$$

where,  $y$  and  $u$  are the output (real GDP in natural logarithm form) and unemployment rate (percentage) in the moment  $t$ , respectively. The trends of the series are  $\bar{y}$  and  $\bar{u}$ , as  $(y_t - \bar{y}_t)$  and  $(u_t - \bar{u}_t)$  denote their cyclical components.  $\alpha_0$  is a constant, while  $\alpha_1$  represents Okun's coefficient.  $\varepsilon_t$  stands for an error term.

The starting point in the wavelet analysis is the selection of wavelet function, the core of this approach. The wavelet function is characterized by zero mean and infinite energy. Out of many such existing functions (e.g., Haar, Morlet, Mexican hat, Paul or Daubechies), the Morlet wavelet confers “a good balance between time and frequency localization” (Grinstead et al., 2004, p. 563).

Two parameters define the Morlet wavelet function  $\psi_0(\eta)$ : the nondimensional ‘time’ parameter  $\eta$  and nondimensional frequency  $\omega_0$ . The function has this form:

$$\psi_0(\eta) = \pi^{-\frac{1}{4}} e^{i\omega_0\eta} e^{-\frac{1}{2}\eta^2}, \quad (2)$$

where  $i$  is  $\sqrt{-1}$ , with a nondimensional frequency set to 6 to satisfy the admissibility condition proposed by Farge (1992).

The next step is called the wavelet transformation, where the time-series are converted into time-frequency by using the Morlet wavelet function. The theory reveals two such transformations: the discrete wavelet transformation (DWT) and continuous wavelet transformation (CWT). If the DWT is typical for time-series treatment (i.e., noise reduction and data compression), the CWT is widely used for feature-extraction purposes (Tiwari et al., 2013). Moreover, Aguiar-Conraria and Soares (2011, p. 647) highlight the utility of CWT in the economic area by showing that “the cross-wavelet analysis could be fruitfully used to uncover time-frequency interactions between two economic time-series.” As we are interested to explore the interaction between two variables, the CWT is clearly more appropriate for our study.

The CWT transformation of a discrete time-series  $\{x_n\}$ , having  $\delta t$  time spacing and scale  $s$  (with  $n=0\dots N-1$ ), is as follows:

$$w_n^x(s) = \frac{\delta t}{\sqrt{s}} \sum_{n'=0}^{N-1} x_{n'} \psi^* \left( (n' - m) \frac{\delta t}{s} \right), \text{ where } m=0, 1, \dots, N-1. \quad (3)$$

Unfortunately, such wavelet transformation is not completely localized in time, generating the so-called 'edge artifacts' (Grinstead et al., 2004, p. 563). Therefore, a Cone of Influence (COI) is introduced in order to deal with this issue that cannot be ignored in the wavelet analysis, as the edge effects might distort the results outside the COI.

The main wavelet method used to analyze the co-movement between cyclical components of unemployment and GDP is the wavelet coherency including phase difference (Torrence and Compo, 1998; Grinstead et al., 2004; Ng and Chan, 2012).

The seminal work in wavelet coherency is the cross-wavelet spectrum (XWT) proposed by Hudgins et al. (1993), which links two time-series,  $x=\{x_n\}$  and  $y=\{y_n\}$ , having this form:

$$W_n^{xy} = W_n^x W_n^{y*}, \quad (4)$$

with  $W_n^x$  and  $W_n^y$  as CWT of  $x$  and  $y$ , respectively.  $|W_n^{xy}|$  shows the cross-wavelet spectrum. Based on Fourier power spectra  $P_f^x$  and  $P_f^y$ , the XWT reveals the confined covariance between two series, for each scale. Herein, the theoretical distribution is:

$$D \left( \frac{|W_n^x W_n^{y*}|}{\sigma_x \sigma_y} < p \right) = \frac{Z_v(p)}{v} \sqrt{P_f^x P_f^y}, \quad (5)$$

where,  $Zv(p)$  is the confidence level of probability  $p$ , while  $pdf$  denotes the square root of the product of two  $\chi^2$  distributions.

Aguiar-Conraria and Soares (2011, p. 649) claim that the XWT has a huge disadvantage, as it “can show strong peaks even for the realisation of independent processes suggesting the possibility of spurious significance tests.” Therefore, an improved version is proposed to fix the issue, namely the wavelet coherency (WTC). The WTC is as follows:

$$R_n(s) = \frac{|S(s^{-1}W_n^{xy}(s))|}{S(s^{-1}|W_n^x|)^{\frac{1}{2}} S(s^{-1}|W_n^y|)^{\frac{1}{2}}}, \quad (6)$$

where,  $S$  is the smoothing operator for both time and scale. The intensity of connection between variables is given by the value of  $R_n$ , the co-movement being stronger as the  $R_n$  increases. A level of significance of 5% is calculated based on the Monte Carlo simulations by following phase randomized surrogate series. The level of 5% is widely used in the time-frequency approach, as the “area of a time frequency plot above the 5% significance level is not a reliable indication of causality” (Grinstead et al., 2004, p. 565). Along the same idea, the authors claim that “it is possible for two series to be perfectly correlated at one specific scale while the area of significant correlation is much less than 5%” (p. 565). The 5% critical level is also suggested in the short- and long-run time-frequency causality approach developed by Breitung and Candelon (2006).

The phase angle or phase difference can be employed based on WTC, with  $\phi_x$  representing the position in the pseudo-cycle of the series for a time series  $x=\{x_n\}$ . For two series  $x=\{x_n\}$  and  $y=\{y_n\}$ , the phase difference  $\phi_{x,y}$  has this form:

$$\phi_{x,y} = \tan^{-1} \left( \frac{\Im\{W_n^{xy}\}}{\Re\{W_n^{xy}\}} \right) \text{ and } \phi_{x,y} \in [-\pi, \pi], \quad (7)$$

where  $\Re$  and  $\Im$  are the real and imaginary part of a complex number, respectively. When  $\phi_{x,y} \in \left[0, \frac{\pi}{2}\right]$ , the series are in phase,  $x$  leading  $y$ . For  $\phi_{x,y} \in \left[-\frac{\pi}{2}, 0\right]$ ,  $y$  leads  $x$ . Conversely, the series are in anti-phase for phase difference  $\pi$  or  $-\pi$ . Herein, for  $\phi_{x,y} \in \left[-\pi, -\frac{\pi}{2}\right]$ ,  $x$  leads  $y$ , while  $y$  leads  $x$  when  $\phi_{x,y} \in \left[\frac{\pi}{2}, \pi\right]$ , respectively.

All methods are computed in Matlab software, based on the code offered by Grinstead et al. (2004), with corrections of Ng and Chan (2012). In this case, the power range goes from blue color (low power) to yellow color (high power), while the phase difference between the two series are indicated through arrows. More precisely, the variables are in phase when the arrows are oriented to the right (positively related), having a cyclical effect. The variables are out of phase when the arrows are pointed to the left (negatively related), registering an anti-cyclical effect.

## Findings

Table A1 (Appendix) presents the descriptive statistics of all considered quarterly variables in their raw levels, while Figures 1 and 2 illustrate the CWT plots and graphics of unemployment and GDP growth rates.

Figure 1: CWT plot and graphic of unemployment growth rate in the US, for the period 1948Q1-2020Q4

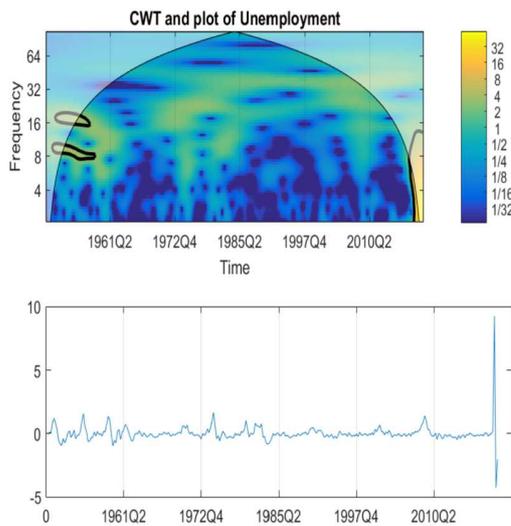
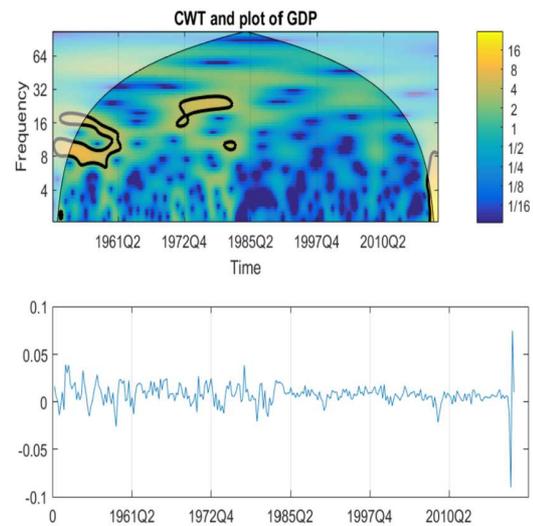


Figure 2: CWT plot and graphic of GDP growth rate in the US, for the period 1948Q1-2020Q4

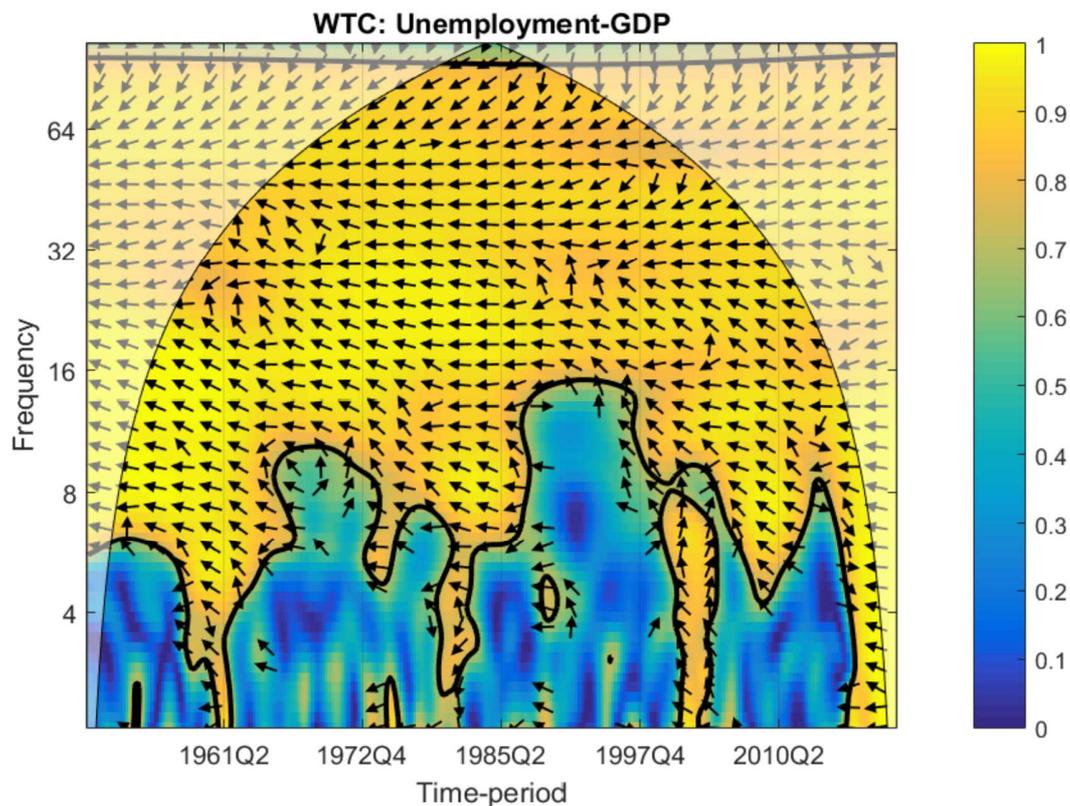


Note: The color code reveals the power intensity of the series, going from blue (low power) to yellow (high power).

It is noteworthy that the CWT plots and graphics unveil important details of the series. Two sequences characterized by high or moderate economic volatility are evidenced over 1948-1983 and 2008-2020 (i.e., the variables in CWT plots show high power), while one sequence with low economic volatility is revealed over 1984-2007, coinciding with the period of Great Moderation (i.e., the variables in CWT plots exhibit low power). The volatility can influence the unemployment in tandem with productivity, impacting the wages as the probability of adverse risks changes.

Figure 3 plots the WTC of cyclical components of unemployment and GDP in the case of the US. The cyclical components are obtained based on the HP filter by extracting the trends from raw series data. As both variables have quarterly frequency, the smoothing parameter lambda is set to 1600.

Figure 3: WTC of the pair 'unemployment - GDP' in US, for the period 1948Q1-2020Q4 (HP cycle components)



Note:

- (1) The thick black contour shows the 5% significance level, while the COI is depicted as a lighted shadow. The power ranges goes from blue color (low power) to yellow color (high power);
- (2) The unemployment is leading when the arrows are oriented to the right and up, while GDP is leading when the arrows are oriented to the right and down. The GDP leading when the arrows are oriented to the left and up, while the unemployment is leading when the arrows are pointed to the left and down.
- (3) The variables are in phase when the arrows are oriented to the right, and are out of phase when the arrows are pointed to the left (negatively related).

The WTC spectrum shows that at **high frequency**, corresponding to a band of scale of up to 4 quarters (short-term), the co-movements between the unemployment and output gaps are rather idiosyncratic. The findings show that Okun's effect is valid in the short-term but only under internal and external economic or pandemic shocks, thereby being rather an 'impulse effect'. Four short episodes are eloquent.

The first episode coincides with the period 1958-1963, where the arrows are pointed to the left and up, the variables being out of phase. Hence, the output gap leads the unemployment gap, with a negative sign. In the US, this sub-period covers an expansion and a recession where internal shocks took place with valid Okun's effect and output gap leading unemployment (e.g., the deficits caused by Vietnam War, the fall of Dow Jones Industrial Average index after only four weeks before its inauguration, the 'rolling adjustment' for many US manufacturing industries and especially the US car makers as there was a starting shift in demand for foreign products).

The second episode is related to 1976-1978. Herein, as the arrows are oriented to the left and down, the unemployment gap leads the GDP one, also with a negative sign. This episode can be

associated with the aftermath of the negative supply shocks that hit the US economy in 1973 and led to a stagflation period: the quadrupling of oil prices as a result of the oil embargo stemming from the Arab-Israeli war of 1973, the sharp increase in food prices as a consequence of a series of crop failures throughout the world, and the end of US wage and price controls in 1973 and 1974 leading workers to push for higher wages.

The third episode coincides with the beginning of the 2000's. In this case, the arrows are pointed to the left and up, showing that the GDP gap negatively drives the unemployment gap. This sub-period corresponds to a series of negative shocks to the aggregate demand that hit the US economy and led to a recession period: the burst of the “tech bubble” in March 2000, the terrorist attacks of September 11, 2001, an increase in financial frictions due to the Enron bankruptcy in 2001 and other corporate accounting scandals, and also the prelude of the housing bubble from 2006-2007.

Finally, the fourth episode is related to the recent COVID-19 pandemic crisis referring to the 2020 year. In this case, the arrows are pointed to the left and up meaning that the output gap negatively leads the unemployment gap. This suggests that the asymmetrical sectoral fall of aggregate supply under pandemic lockdown restrictions compressed the economy thus generating unemployment. Unfortunately, this result should be interpreted with caution as half of the episode is outside the COI, with the estimation suffering 'edge effects' (i.e., the quality of result can be strongly distorted).

At **medium frequency**, associated with a band of scale between 5 and 39 quarters (medium-term), the arrows are pointed to the left and up. This denotes that the output gap negatively leads the unemployment gap, with Okun's law effect being registered at the business cycle level. Moreover, the output clearly proves the existence of cyclical unemployment (i.e., Keynesian) in the medium-term, highlighting the crucial role of aggregate demand.

By following the volatility characteristics of the series, interesting results are revealed, partially being in line with Ho (2014). More precisely, the action of Okun's effect seems to disappear at 4 to 16 quarters band of scale over the period of Great Moderation (i.e., low volatility). Further, the effect is observed at the top of a medium band of scale, for 16-39 quarters. Otherwise, over the periods 1948-1984 and 2007-2020, characterized by higher economic volatility, the effect is stronger only for more than 12 quarters band of scale. In other words, the economic volatility influences the stability of Okun's effect rather than its intensity in the medium-term.

At **low frequency**, corresponding to a band of scale of more than 40 quarters (long-term), the variables are still out of phase, the arrows being oriented to the left but down. This denotes that the unemployment gap negatively leads the output gap validating, once more, Okun's law. Additionally, the finding indicates the existence of classical unemployment suggesting that the real wages are above the market-clearing wage with a tendency to long-run equilibrium.

The results confirm both directions of Okun's law co-movement but two interesting findings are different with respect to the existing ones: the sensitivity of 'unemployment-output' interaction over different sub-periods of time, and the stability of related effect. As the unemployment rate has a particular characteristic (Apergis and Arisoy, 2017) while the investigated sample has different volatility episodes, two additional connections are called to support the general results: one related to 'participation rate - GDP' and another focused on 'productivity - GDP' (Benigno et al., 2015). Their related WTC and WC plots obtained, based on the HP filter, are presented in Figures A1-A4 (Appendix).

The WTC and WC of 'participation - output' co-movement show that Okun's law is observed only in the medium-term, output positively running participation rate (i.e., arrows are pointed to the right and down in the WTC, while yellow color denotes the pro-cyclical co-movement in the WC). When the economy is in expansion, better job prospects encourage more people to enter into the labor force. The Keynesian unemployment is reinforced being robust to participation rate. This also suggests an inverse connection between unemployment and participation, being in line with Apergis and Arisoy (2017). No co-movements are registered at high- and low-frequencies. Those differences come from the content of the labor indicators that are used. The unemployment rate considers officially registered unemployed persons plus those who receive unemployment benefits (i.e., no other jobless persons included), while participation rate captures both employed and unemployed persons. This shows that Okun's effect is very sensitive to the selection of labor indicators called to capture the jobless phenomenon.

Finally, the WTC and WC plots of the 'productivity-output' pair indicate interesting results. Herein, the productivity is positively related to output on both short- and medium-terms, the arrows being pointed to the right and up. When the economy is expanding, the producers make their employees work harder by increasing their hours. Curiously, the sign of co-movement slightly changes over the long-term for more than 40 quarters band of scale. Herein, the arrows are oriented to the left and up, indicating that the output and productivity have a counter-cyclical effect.

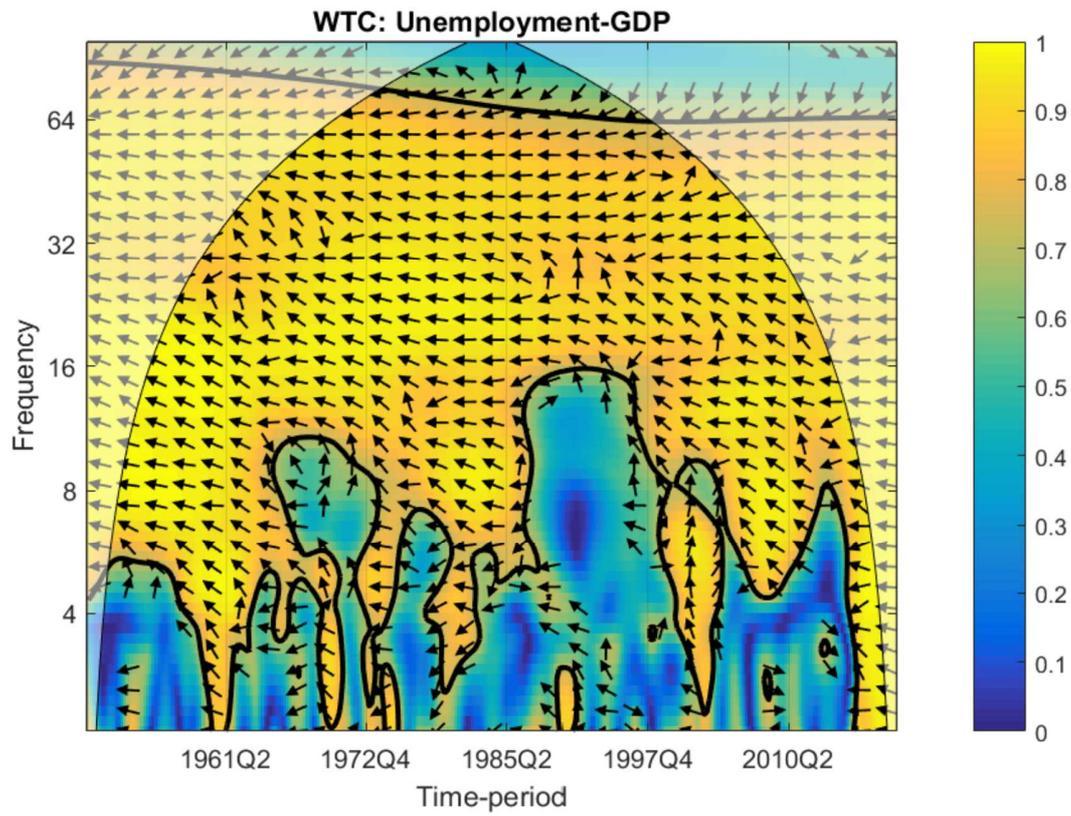
All these results are in accord with Moosa (1999) and Aguiar-Conraria et al. (2020), as Okun's effect is strongly impacted by the business cycle. Unlike Aguiar-Conraria et al. (2020), our approach puts into evidence Okun's law also in short-term but under strong turbulences. The findings partially validate Harris and Silverstone (2001), and Cuaresma (2003), who claim that the effect is not stable over time. Such differences come from both using time-span frequency and following methodology. The findings are in line with Barreto and Howland (1994) regarding directions of causality, additionally offering more insights regarding Keynesian and classical unemployment. Finally, the outputs do not confirm Guisinger et al.'s (2018) findings as not only the output is considered as dependent variable but also unemployment.

## **Robustness check**

The robustness check follows: (1) an alternative dataset constructed by capturing the cyclical components of both time-series as 'year-on-year' quarterly growth rate; (2) an alternative time-frequency tool proposed by Rua (2010), namely the wavelet cohesion (WC); and (3) a scenario with monthly frequency by routinizing all considered cross-wavelet tools.

(1) The alternative dataset is constructed in order to remove the trend component of both time-series by considering their 'year-on-year' quarterly growth rate. Related WTC plot is shown in Figure 4.

Figure 4: WTC of the pair 'unemployment - GDP' in US, for the period 1948Q1-2020Q4 ('year-on-year' quarterly growth rate)



Note:

- (1) The thick black contour shows the 5% significance level, while the COI is depicted as a lighted shadow. The power ranges goes from blue color (low power) to yellow color (high power);
- (2) The unemployment is leading when the arrows are oriented to the right and up, while GDP is leading when the arrows are oriented to the right and down. The GDP leading when the arrows are oriented to the left and up, while the unemployment is leading when the arrows are pointed to the left and down.
- (3) The variables are in phase when the arrows are oriented to the right, being out of phase when the arrows are pointed to the left (negatively related).

The plot shows that the WTC constructed based on 'year-on-year' quarterly growth rates reveals almost the same results as in the case of WTC with the HP dataset. Up to 4 quarters band of scale, all four episodes, are clearly revealed at high frequency: 1958-1963, with both expansion and recession sequences; oil shocks from 1970; recession of 2000's; and pandemic disease from 2019-2020, with limitation given by the COI's 'edge effects'. Further, at 5-39 quarters band of scale, the arrows are oriented to the left and up, validating the idea of cyclical unemployment as the output drives unemployment. Finally, for more than 40 quarters band of scale, unemployment drives output, the classical unemployment being reinforced (i.e., the arrows are pointed to the left and down). The disappearing Okun's effect over Great Moderation is also reinforced over 1984-2007.

(2) The alternative tool is the wavelet cohesion (WC) proposed by Rua (2010), having as a starting point the contribution of Croux et al. (2001). In this light, Rua (2010) proposes a co-movement measure  $\rho_{x_n y_n}$  as a real number on  $[-1, 1]$  by using only the real part of wavelet cross-spectra, with this form:

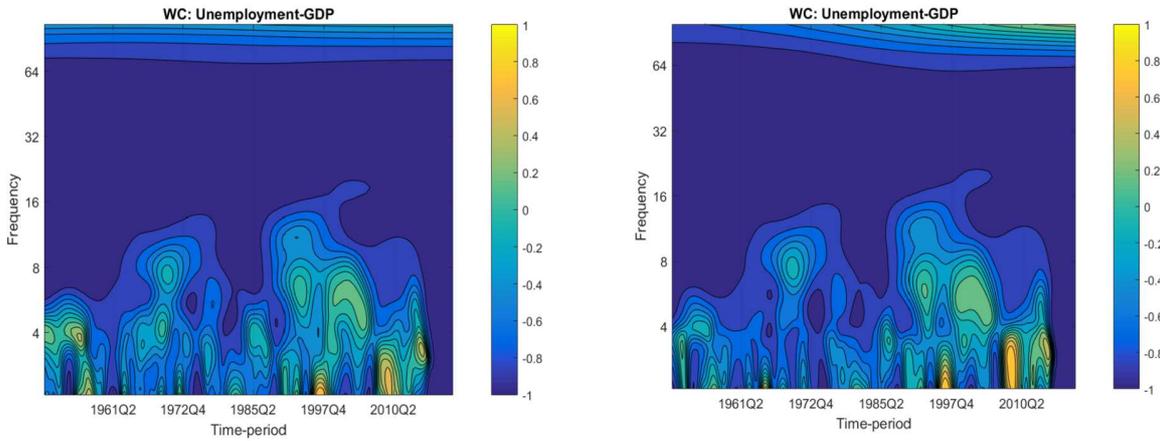
$$\rho_{x_n y_n} = \frac{\Re(W_n^x W_n^y)}{\sqrt{|W_n^x|^2 |W_n^y|^2}} \quad (8)$$

Therefore, the WC does not capture only the positive correlations between variables, but also those negative ones, being a good alternative to WTC regarding the phase-difference direction status of the series.

The WC spectra generated based on both the HP filter and 'year-on-year' quarterly growth rate series are plotted in Figures 5 and 6.

Figure 5: WC of the pair 'unemployment - GDP' in US, for the period 1948Q1-2020Q4 (HP cycle components)

Figure 6: WC of the pair 'unemployment - GDP' in US, for the period 1948Q1-2020Q4 ('year-on-year' quarterly growth rate)



Note: The color code reveals the intensity of correlations, going from blue color (positive correlation) to yellow color (negative correlation).

Both plots are almost identical, offering similar findings when compared with the WTC ones. Okun's effect is clearly evidenced especially over medium and high frequency, reinforcing the results of WTC's plots. Therein, the color is blue, indicating that the unemployment and output exhibits an anti-cyclical co-movement. Unfortunately, no information about direction of co-movement is offered by the WC tool. At high frequency, up to 4 quarters band of scale, the blue color is also present but alternates in intensity, validating the idiosyncratic characteristic of co-movement in the short-term.

(3) The monthly dataset includes the monthly unemployment rate communicated by FRED (2021), while the GDP is estimated by linearly converting to monthly frequency the official quarterly GDP offered by FRED (2021). Their descriptive statistics are illustrated in Table A2 (Appendix). Both variables are alternatively used as HP cyclical components and 'year-on-year' monthly growth rate. As the white-noise component of the series can induce disturbance in time-frequency analysis (Rua, 2010; Mutascu, 2018), especially for variables expressed in growth rate, this property is checked via the Portmanteau test for white-noise. The test considers both HP and growth rate variables, with their quarterly and monthly forms of frequency, as Table A3 (Appendix) shows. The results clearly prove that all tested variables do not have a white-noise component (i.e., the null hypothesis that the variables that follow a white-noise process can be rejected for all levels of significance).

Finally, the WTC and WC findings obtained based on monthly variables both in their HP cyclical component and 'year-on-year' monthly growth rate forms are presented in the Figures 7-10.

Figure 7: WTC of the pair 'unemployment - GDP' in US, for the period 1948M1-2020M12 (HP cycle components)

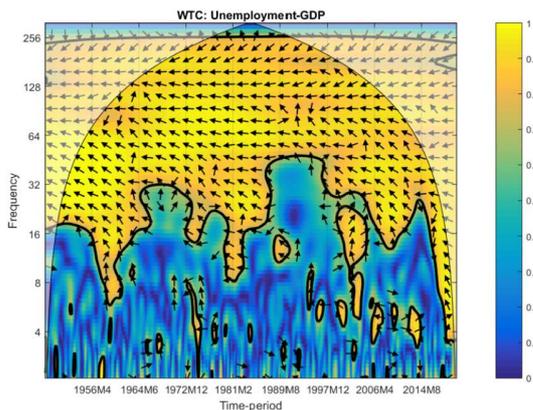
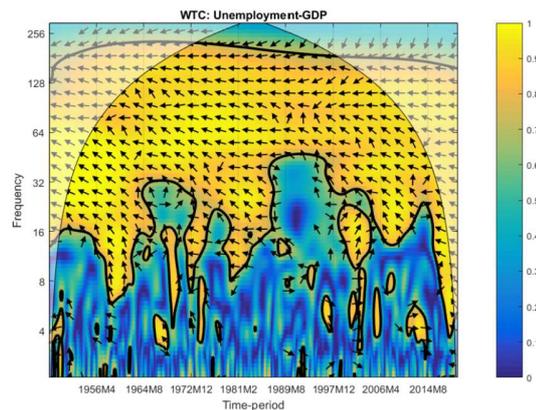


Figure 8: WTC of the pair 'unemployment - GDP' in US, for the period 1948M1-2020M12 ('year-on-year' monthly growth rate)



Note:

- (1) The thick black contour shows the 5% significance level, while the COI is depicted as a lighted shadow. The power ranges goes from blue color (low power) to yellow color (high power);
- (2) The unemployment is leading when the arrows are oriented to the right and up, while GDP is leading when the arrows are oriented to the right and down. The GDP leading when the arrows are oriented to the left and up, while the unemployment is leading when the arrows are pointed to the left and down.
- (3) The variables are in phase when the arrows are oriented to the right, being out of phase when the arrows are pointed to the left (negatively related).

Figure 9: WC of the pair 'unemployment - GDP' in US, for the period 1948M1-2020M12 (HP cycle components)

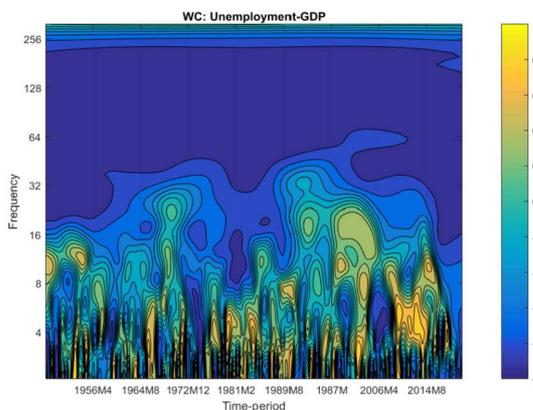
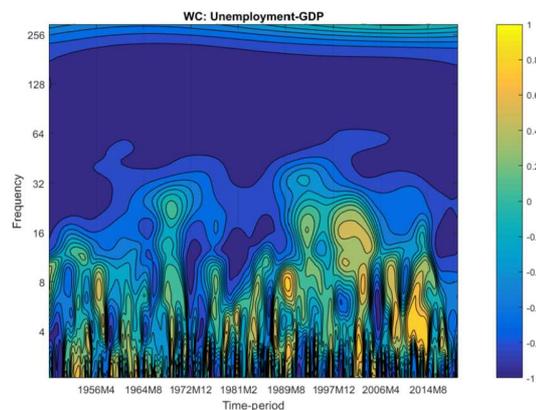


Figure 10: WC of the pair 'unemployment - GDP' in US, for the period 1948M1-2020M12 ('year-on-year' monthly growth rate)



Note: The color code reveals the intensity of correlations, going from blue color (positive correlation) to yellow color (negative correlation).

The plots indicate that the results remain robust to dataset frequency switch, the Okun's effect with monthly dataset being clearly reinforced especially over both medium- and long-terms. As in the previous cases, the findings exhibit an idiosyncratic co-movement in the short-term. According to Hamilton (2018), the HP filter can induce spurious dynamic differences between the filtered values in the middle with respect to those at the end of sample. In order to check if this issue alters the monthly result, a set of WTC and WC plots is employed by dropping the year 2020, as Figures A5-A6 (Appendix) show. As the plots show, no influence on co-movement is observed by removing the 2020 year, the estimations remaining robust for the rest of period.

## Conclusions

The paper explores the interaction between the cyclical components of unemployment and output in the US by using the wavelet tool. The study covers the period 1948Q1-2020Q4. The main findings bring new insights regarding the direction and duration of co-movement between the cyclical components of unemployment and output.

The short-term shows that Okun's effect is valid only under socio-economic shocks, being rather an 'impulse effect'. There are two sub-effects. The first sub-effect invokes *internal shocks*, when the GDP negatively drives unemployment. The negative demand shock leads to a decline in household and business spending, causing the output and employment to adjust downward, and the unemployment to rise above its natural rate. Noteworthy is that the pandemic crisis seems to be 'endogenized' despite the fact that it is an external shock. Whatever the case, the pandemic episode should be interpreted with caution. The second sub-effect is related to *external shocks*, the unemployment negatively driving the output. In this case, the oil/energy negative supply shocks lead to an increase in the price level-via production costs. Workers push for higher wages that outpace productivity gains, causing a rise of real wages. This generates higher unemployment, eventually reducing output.

In the medium-term, the GDP negatively drives unemployment at the business cycle, proving the existence of cyclical unemployment. The Keynesian unemployment is confirmed. During recessions, the fall of aggregate demand reduces production and 'slowly' compresses wages, generating unemployment. More interesting is that the co-movements seem to be sensitive to economic volatility, disappearing at low volatility because of the probability of adverse risks that can push the wages and, further, the unemployment is reduced.

In the long-term, the unemployment negatively runs GDP, suggesting the existence of classical unemployment. The rise of real wages above the market-clearing ones (e.g., increase of minimum wage, pressure of labor union) provokes unemployment, reducing growth. Herein, the fall of output is accompanied by a slight increase of productivity as the input declines faster than the reducing trend of GDP.

Regarding *policy implications*, the US policymakers should adapt their interventions, as our results clearly demonstrate that there are short-, medium- and/or long-term approaches.

In the short-term facing internal shocks, the intervention should involve expansionary fiscal policy and an easing of monetary policy in order to stimulate aggregate demand and production, absorbing the 'temporary' unemployment. Two kinds of policy responses are possible for external shocks, depending on the stabilization objectives in the short-term. If the stabilization target is inflation, the monetary policy should be implemented by raising the real rate of interest although it temporarily causes higher unemployment. Otherwise, if policymakers prefer to stabilize

economic activity rather than inflation, they should ease the monetary policy by lowering the real interest rate assuming inflation in the short-term.

In the medium-term, the economic policy should act on the aggregate demand (i.e., demand approach). During recessions, an expansionary policy is required that would involve tax cuts, public spending programs and accommodative monetary supply. Those measures stimulate aggregate demand, increase production, slowly increase wages and absorb unemployment. The impact on inflation of such demand-oriented policies deserve special attention during an expansion period because the positive impact may be gradually offset by wages increases pushed by workers.

In the long-term, the classical unemployment suggests no intervention (i.e., supply approach), as the US economy presents a strong self-corrective mechanism. If real wages are initially above the market-clearing level, this generates unemployment and more slack in the economy, which then relaxes the pressure on wages, reducing the cost for suppliers. Further, this expands production and contributes to the lowering of unemployment.

The main research limit is due to the lack of controls to isolate the co-movement of interest variables, while the wavelet tools used in this study are par excellence bi-variate ones. As a further investigation, an extension of this study could be carried out by taking into account control variables with the implementation of multiple wavelet coherency (MWC) and partial wavelet coherency (PWC) proposed by Mihanovic et al. (2009).<sup>1</sup> [Such an approach allows the isolation of the bi-variate action of co-movement between unemployment and output by showing how this evolves across frequencies under different determinants. More precisely, the tools offer important information about the sensitivity of Okun's effect under different economic stimuli which would help to better design adequate economic policies.](#)

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<sup>1</sup> No software code is available in this moment for more than two control determinants.

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## Appendix

Table A1: Summary statistics of main raw variables (quarterly frequency)

	Unemployment rate (%)	Real GDP (Billions of Chained 2012 Dollars)	Participation rate (%)	Productivity index
Mean	5.766667	8906.485	62.86781	3.248303
Median	5.55	7649.364	63.05	3.3192
Maximum	13.06667	19253.96	67.3	31.78044
Minimum	2.566667	2086.017	58.5	-47.0682
Std. Dev.	1.699597	5213.925	2.977035	5.687126
Skewness	0.805092	0.423637	-0.04695	-1.9877
Kurtosis	3.812709	1.831392	1.480558	25.1861
Jarque-Bera Probability	39.58044 0	25.34946 0.000003	28.19652 0.000001	6180.994 0
Sum	1683.867	2600694	18357.4	948.5044
Sum Sq. Dev.	840.5911	7.91E+09	2579.057	9411.932
Observations	292	292	292	292

Table A2: Summary statistics of raw variables (monthly frequency)

	Unemployment rate (%)	Real GDP (Billions of Chained 2012 Dollars)
Mean	5.764531	8902.988
Median	5.6	7649.35
Maximum	14.8	19254
Minimum	2.5	2086
Std. Dev.	1.708285	5197.726
Skewness	0.83846	0.423323
Kurtosis	4.087239	1.830918
Jarque-Bera Probability	145.4537 0	75.87642 0
Sum	5038.2	7781211
Sum Sq. Dev.	2547.62	2.36E+10
Observations	876	876

Table A3: Portmanteau test for white noise

Variable	Portmanteau test for white noise (quarterly frequency)	Portmanteau test for white noise (monthly frequency)
Unemployment rate (HP cycle component)	Q-stat.= 391.8514 Prob. = 0.0000	Q-stat.= 2290.5957 Prob. = 0.0000
GDP (HP cycle components)	Q-stat.= 498.9567 Prob. = 0.0000	Q-stat.= 4056.5268 Prob. = 0.0000
Unemployment rate (‘year-on-year’ monthly growth rate)	Q-stat.= 348.3150 Prob. = 0.0000	Q-stat.= 3394.7493 Prob. = 0.0000
GDP (‘year-on-year’ monthly growth rate)	Q-stat.= 356.7404 Prob. = 0.0000	Q-stat.= 3982.9140 Prob. = 0.0000

Figure A1: WTC of the pair 'participation - GDP' in US, for the period 1948Q1-2020Q4 (HP cycle components)

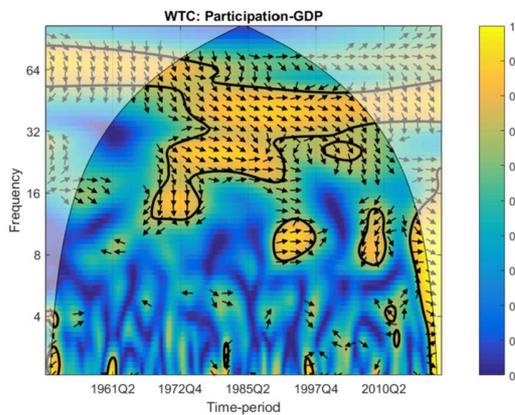
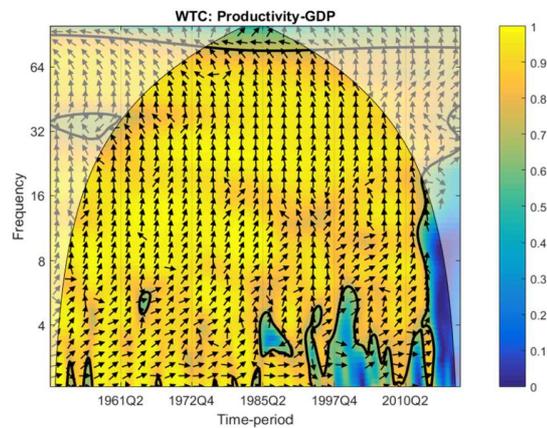


Figure A2: WTC of the pair 'productivity - GDP' in US, for the period 1948Q1-2020Q4 (HP cycle components)



Note:

- (1) The thick black contour shows the 5% significance level, while the COI is depicted as a lighted shadow. The power ranges goes from blue color (low power) to yellow color (high power);
- (2) The participation/productivity is leading when the arrows are oriented to the right and up, while GDP is leading when the arrows are oriented to the right and down. The GDP leading when the arrows are oriented to the left and up, while the participation/productivity is leading when the arrows are pointed to the left and down.
- (3) The variables are in phase when the arrows are oriented to the right, being out of phase when the arrows are pointed to the left (negatively related).

Figure A3: WC of the pair 'participation - GDP' in US, for the period 1948Q1-2020Q4 (HP cycle components)

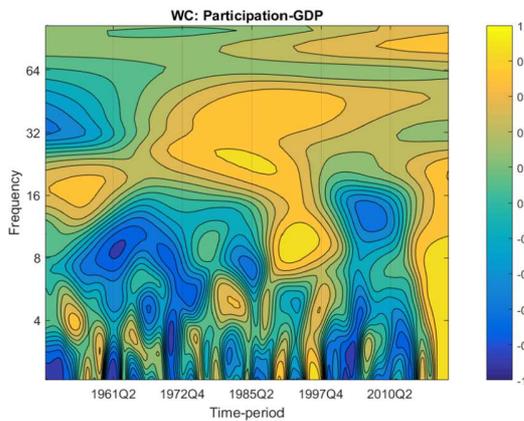
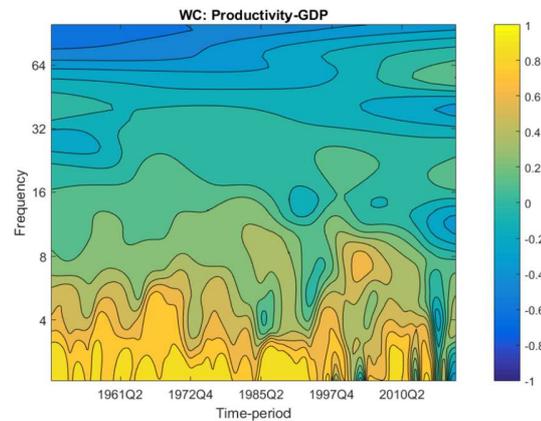


Figure A4: WC of the pair 'productivity - GDP' in US, for the period 1948Q1-2020Q4 (HP cycle components)



Note: The color code reveals the intensity of correlations, going from blue color (positive correlation) to yellow color (negative correlation).

Figure A5: WTC of the pair 'unemployment - GDP' in US, for the period 1948M1-2019M12 (HP cycle components)

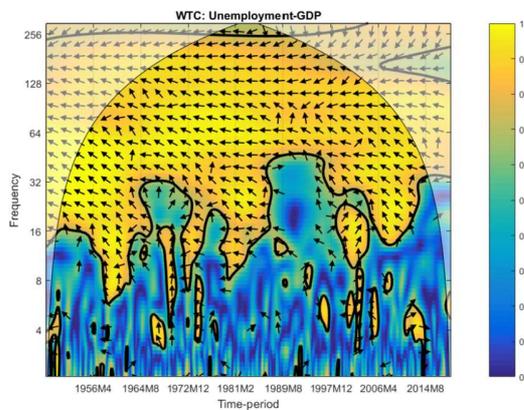
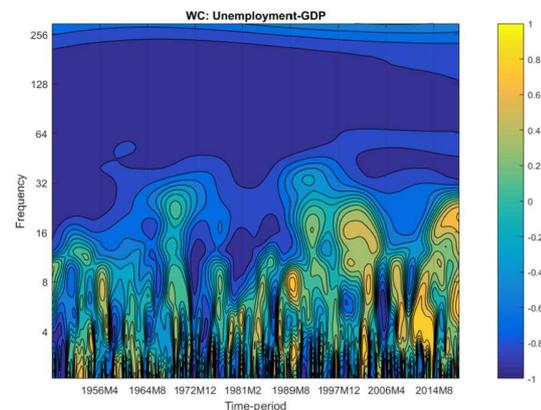


Figure A6: WC of the pair 'unemployment - GDP' in US, for the period 1948M1-2019M12 (HP cycle components)



Note:

I. WTC:

- (1) The thick black contour shows the 5% significance level, while the COI is depicted as a lighted shadow. The power ranges goes from blue color (low power) to yellow color (high power);
- (2) The unemployment is leading when the arrows are oriented to the right and up, while GDP is leading when the arrows are oriented to the right and down. The GDP leading when the arrows are oriented to the left and up, while the unemployment is leading when the arrows are pointed to the left and down.
- (3) The variables are in phase when the arrows are oriented to the right, being out of phase when the arrows are pointed to the left (negatively related).

II. WC: The color code reveals the intensity of correlations, going from blue color (positive correlation) to yellow color (negative correlation).