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# A Monetary Model of Growth with Limited Foresight\*

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## Abstract

Rational expectations are often questioned in light of their overly demanding assumptions. Thus, an increasing literature introduces some form of bounded rationality. In this paper, we study real and monetary growth models with agents endowed with limited foresight. Accordingly, in each period, economic plans extend only for a limited number of periods and are reformulated in each subsequent date. We show that limited foresight may lead to capital under-investment and be thus growth-detrimental. However, by relaxing progressively myopia, the economy converges to the Perfect Foresight equilibrium. We prove the existence of a monetary Balanced Growth Path (*BGP*) beside the non monetary one and compare it with the outcome obtained under perfect foresight. We also perform a stability analysis and show that the monetary *BGP* is globally unstable (stable) while the non monetary one is globally stable (unstable) when money is positive (negative). Finally, we identify the optimal monetary policy maximizing welfare. Limited foresight, in contrast to a widespread literature, thus restores monetary equilibria even in absence of limited participation, financial frictions and borrowing constraints.

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# 1 Introduction

The recent crises experienced by the world economy are several times the result of unpredictable shocks. Most of the time, even their consequences are unknown and may require policy responses totally new. Crisis and economic recovery are usually analyzed within dynamic general equilibrium models with agents endowed with perfect foresight and thus able to forecast, at least in terms of probabilistic distributions derived from past observations. Indeed, consumers, firms and policy makers may set contingent plans, if markets are complete, for a potentially infinite number of periods. Unfortunately, the traditional risk assessment analysis cannot apply to recent events, as the pandemic, the intense geopolitical tensions, and the profound climate change. Such events seem indeed not to be easily reducible to probability distributions, and to fall rather than into the "risk" and "uncertainty" category, into that of "ignorance". In the presence of ignorance, in fact, agents neither know all the possible states of the world nor the probabilities associated with them. It follows the extreme difficulty to adopt traditional stochastic optimization techniques.

Several attempts to tackle such problematic issues have been proposed in a recent literature. Some contributions introduce indeed bounded rationality as the outcome of limited foresight, for example by assuming agents to form expectations using adaptive learning or to do not pay full attention to some variables in the budget constraint when maximizing utility, because information is costly (Evans and Honkapohja, 2012; Gabaix, 2020). Other studies (Lovo and Polemarchakis, 2010; Woodford, 2019; Woodford and Xie, 2022), by contrast, model bounded rationality by supposing agents, despite having an infinite life horizon in front of them, to behave as they were short-lived. i.e. to formulate consumption and investment plans only for a limited number of periods ahead and to revise these plans in each subsequent date, by taking into account the arrival of new information, included, may be, unpredictable events.

Lovo and Polemarchakis (2010) assume myopic behavior within a model with neoclassical production function, an exogenously given endowments stream, and a constant money supply. They prove the existence of monetary equilibrium and its local (in)determinacy. Furthermore, they compare their model with standard *OLG* ones, by assuming short planning horizons. Their results, however, rely significantly on factors substitutability and on specific hypotheses on the endowment stream. In two related papers, Woodford (2019) and Woodford and Xie (2022) formalize limited foresight within a model characterized by a stochastic environment, absence of capital accumulation, articulated learning processes and specific monetary policies aimed at pegging the interest rate. Within such stationary economy, Woodford (2019) frames limited foresight by assuming agents, in each date, to formulate their plans in

terms of consumption and savings only for a finite number of periods. However, Woodford (2019) assumes assets held in the last period of the planning horizon not equal to zero, in view of a specific value function evaluating positively such stock. The value function, however, does not reflect the true state the economy is actually experiencing, but an hypothetical stationary state, that agents have come to expect after a hypothetical and long enough learning process.

In this paper we apply this type of limited foresight or "myopia" to a monetary  $Ak$  growth model in the spirit of Lovo and Polemarchakis (2010). Money is introduced without any specific motivation of its demand and thus is to be viewed as a purely speculative asset. As it is well known, in infinite horizon models, as stressed by Miao (2014), Santos and Woodford (1997), and Martin and Ventura (2018), conditions under which bubbles exist are relatively fragile and require financial frictions (Scheinkman and Weiss, 1986; Le Van and Pham, 2016; Bosi et al., 2022) or borrowing constraints (Woodford, 1986; Grandmont et al., 1998). By contrast, in economies free from such market imperfections, monetary equilibria are ruled out or money turns out to be completely neutral.

By assuming limited foresight, on the contrary, we prove the existence of monetary equilibria and money non-neutrality even in absence of limited participation, financial frictions and borrowing constraint. Let us notice that also Lovo and Polemarchakis (2010) obtain monetary equilibria. Nevertheless, we improve their analysis in several important directions. First, our  $Ak$  model allows to analyze the impact of myopia and money on growth. Second, it enables us to compute closed form solutions. Third, we can evaluate monetary policy. Fourth, our framework is fitted for performing a complete global stability analysis as well as a welfare one. Fifth, in view of the explicit solutions obtained, we can compare the growth rates of the real and monetary economies with limited foresight with those obtained in the rational expectation ones. Finally, we can evaluate the effects on growth, monetary equilibria and stability, obtained by relaxing the degree of myopia and letting the models converge toward the perfect foresight one.

We obtain a set of new and innovative results. First, we demonstrate that in the real model, the growth rate of the economy increases as the degree of myopia is relaxed, provided that agents do not place too much importance on the utility of the last period of their short optimization horizon. When myopia is completely removed, the system converges to the perfect foresight model. Secondly, we prove the existence of monetary equilibria with both positive and negative money. The only condition for this is that the monetary policy implemented by the Central Bank consists of a constant contraction of the money supply. We also compare and rank the  $BGP$ s obtained in the real and monetary economies with limited foresight with that corresponding to Perfect Foresight Equilibrium ( $PFE$ ). Then we perform

a global stability analysis of monetary equilibrium and prove that when money is positive, it is globally unstable but there arises an alternative and globally stable non-monetary long run equilibrium. On the other hand, when money is negative, the monetary equilibrium becomes globally stable while the non monetary one globally unstable. Nevertheless, under any configuration, global indeterminacy generically is bound to prevail. Such findings reinforce the typical criticism moved by, e.g., Grandmont et al. (1998), against the purely linear analysis carried out in standard *RBC* and *DSGE* models: the local uniqueness of equilibrium does not ensure, from a global perspective, the existence of alternative attractors. Finally, since the rate of growth of real balances drives that of the whole system, we are able to find the optimal monetary policy maximizing welfare. We prove that such policy corresponds to the Friedman's rule and requires to withdraw money at a factor equal to the growth rate to interest rate ratio.

The remainder of the paper is organized as follows. In Section 2 we present the real economy with limited foresight. Section 3 is devoted to the study of the monetary model and to the comparison of the growth rates of the different economies studied. In Section 4 we carry out a welfare analysis, while Section 5 concludes the paper.

## 2 A real model with limited foresight

In this Section we formalize limited foresight and myopic behavior within a simple real economy displaying unbounded growth and characterized by the presence in dates  $t = 0, 1, 2, \dots$  of a unique representative agent. Before explaining to what extent we refer as to limited foresight and how we formalize it, let us previously describe the fundamentals of the economy, i.e. technology and preferences. To keep things as simple as possible, we focus on a very basic growth model allowing for closed form solutions. To this end, we consider a standard one-sector optimal *Ak* model where capital  $k$  is the unique productive factor available in the economy and used to produce both consumption  $c$  and investment  $I$  good. The *Ak* model typically assumes that when people accumulate capital, learning by doing phenomena generate increasing returns that tend to raise the marginal product of capital, thus offsetting its tendency to diminish when capital grows. Celebrated papers introducing technologies whose reduced forms are of *Ak* type are those of Romer (1986), Lucas (1988), Barro (1990), Jones and Manuelli (2005). In our model, by contrast, we do not rely on any external effect and assume a private production function linear in capital, where the marginal product of capital is constant. Such assumption entails that, in absence of further distortions, the decentralized economy mimics perfectly the social planner solution. More specifically, we

assume that in period  $t$  the resource constraint has the form:

$$c_t + I_t = Hk_t \tag{1}$$

where  $H > 0$  is a technological scale parameter measuring capital productivity. The investment good produced in time  $t$  increases the stock of capital available in the subsequent period  $t + 1$  net of the depreciation rate  $\phi \in [0, 1]$ . The law of capital accumulation can therefore be written as

$$k_{t+1} = I_t + (1 - \phi) k_t \tag{2}$$

i.e. the resource constraint in period  $t$  boils down to:

$$k_{t+1} + c_t = Ak_t \tag{3}$$

where  $A \equiv H + (1 - \phi)$ . The representative agent per-period utility function is assumed to be of the *CRRA* form in order to be compatible with balanced growth:

$$u(c_t) = c_t^{1-\varepsilon} / (1 - \varepsilon) \tag{4}$$

with  $\varepsilon > 0$  the inverse of the intertemporal elasticity of substitution of consumption.

We now turn attention to individual behavior under the hypothesis that the representative agent is not endowed with perfect foresight. For sake of concreteness, we model limited foresight following Lovo and Polemarchakis (2010), that assume myopic households. This implies that, despite having an infinite horizon of life in front of them, agents limit themselves to formulate plans in terms of consumption and investment in every period  $t$  only for a finite number of periods ahead. We will make this limited horizon of time to extend increasingly from one, which corresponds to extreme myopia, to infinity, which coincides to the case of agents endowed with perfect foresight. In particular, we will denote with  $n = 1, 2, \dots$ , the planning horizon. In other words,  $n$  can be reinterpreted as the inverse of the degree of myopia: in any period  $t$  agents chose how much to consume and invest only for the following  $n$  periods ahead, i.e. in  $t, t+1, \dots, t+n$ . It follows that at time  $t+n$ , for a myopic agent there is no rationale for further investing and the whole income is therefore entirely devoted to finance consumption. Agents thus solve a finite maximization problem, given the initial condition represented by the capital available at time  $t$ ,  $k_t$ , and subject to the final condition requiring zero capital at the end of period  $t+n$ , i.e.  $k_{t+n+1} = 0$ . This makes it possible to derive the entire sequence of consumption and investment plans from  $t$  up to  $t+n$ , as a function of the initial capital stock  $k_t$ , that represents the outcome of the individual decisions undertaken in the previous period  $t-1$ . Nevertheless, notice that at  $t+1$ , agents realize that they have

in front of them not only  $n - 1$  residual periods of life, but again  $n$ ; accordingly, they will reformulate their plans in terms of consumption and investment for the following  $n$  periods, i.e. up to date  $t + 1 + n$ , taking the previously programmed capital  $k_{t+1}$  as the new initial condition. This procedure will then be iterated infinitely many times. Notice that plans formulated in such way are generically time-inconsistent, except for initial consumption and investment, since they determine the state of the economy in the subsequent maximization program. As we will show in the sequel, this simple specification for the economy makes it possible to find the explicit law for the capital accumulation. Such law will be of the type  $k_{t+1} = f^n(k_t)$ , since it will depend on the degree  $n$  of myopia. By relaxing thus  $n$  from one to infinite, we will obtain a well defined sequence of functions, whose features and convergence will be the main object of our discussion.

Let's now solve the maximization problem of the representative agent in time  $t$ , given the initial endowment of capital  $k_t$  and under an (inverse) degree  $n$  of myopia. Such problem can be written as:

$$\max_{\{c_{t+j}, k_{t+j+1}\}_{j=0}^n} \sum_{j=0}^{n-1} \beta^{t+j} \frac{c_{t+j}^{1-\varepsilon}}{1-\varepsilon} + \delta \beta^{t+n} \frac{c_{t+n}^{1-\varepsilon}}{1-\varepsilon} \quad (5)$$

subject to constraints

$$c_{t+j} + k_{t+1+j} = r_{t+j} k_{t+j} + (1 - \phi) k_{t+j} \quad (6)$$

for  $j = 0, \dots, n - 1$  and

$$c_{t+n} = r_{t+n} k_{t+n} + (1 - \phi) k_{t+n}, \quad (7)$$

where  $r_{t+j}$  is the real rental price of capital and  $\delta > 0$  is a positive weight according to which agents evaluate last period utility, to be viewed as a proxy of all future utilities (Lovo and Polemarchakis, 2010).<sup>1</sup> By integrating backward constraints (6) and (7) and setting  $R_{t+j} \equiv r_{t+j} + (1 - \phi)$  the gross real interest rate, we can write the intertemporal budget constraint in the form:

$$c_t + \frac{c_{t+1}}{R_{t+1}} + \frac{c_{t+2}}{R_{t+1}R_{t+2}} + \dots + \frac{c_{t+n}}{R_{t+1}R_{t+2}\dots R_{t+n}} = R_t k_t \quad (8)$$

The individual program can be thus reformulated as to maximize (5) subject to (8). Solving it simply requires to introduce the Lagrangian and to compute the first order conditions which are immediately obtained as:

$$c_{t+j-1}^{-\varepsilon} = \beta c_{t+j}^{-\varepsilon} R_{t+j} \quad (9)$$

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<sup>1</sup>Lovo and Polemarchakis (2010) assume  $\delta > 1$ . We relax this hypothesis since this does not influence the mechanism yielding to the results.



for  $j = 1, \dots, n - 1$  and

$$c_{t+n-1}^{-\varepsilon} = \beta \delta c_{t+n}^{-\varepsilon} R_{t+n} \quad (10)$$

together with the budget constraint (8). The time inconsistency of individual plans is immediately apparent once one observes, e.g., that the capital holding  $k_{t+n+1}$  planned in period  $t$  is zero, while it will be positive when planned in  $t + 1$ . In the limited foresighted economy, the weaker concept of *perfect foresight spot equilibrium (PFSE)* must now be applied (Lovo and Polemarchakis, 2010). According with this definition, while the first order conditions hold from  $t$  up to  $t + n$ , market clearing is required only at the time of optimization, because future markets do not exist. In such way, individuals can borrow or lend for one period the nominal asset and capital. It follows that an equilibrium is defined by the final good market clearing and the capital market clearing at current time  $t$ :

**Definition 1.** *A perfect foresight spot equilibrium (PFSE) for the economy is a sequence of rental prices of capital  $\{r_i\}_{i=0}^{\infty}$  such that, in every date  $t$ , the spot markets for capital and consumption are in equilibrium:*

$$r_t = A - (1 - \phi) \text{ and } k_{t+1} = (r_t + 1 - \phi) k_t - c_t. \quad (11)$$

In other words, in a PFSE, individuals make plans in terms of future consumption on the basis of their expectation about the spot price that will be observed in the future. It is then immediate to verify that the usual Perfect Foresight Equilibrium (PFE) is also PFSE but not conversely.

By combining the first order conditions (8), (9) and (10) with the equilibrium ones (11), one can express consumption choices as a function of initial consumption  $c_t$ , i.e.

$$c_{t+j} = \left( (A\beta)^{\frac{1}{\varepsilon}} \right)^j c_t, \quad (12)$$

for  $j = 0, \dots, n - 1$  and

$$c_{t+n} = (A\beta)^{\frac{1}{\varepsilon}} \delta^{\frac{1}{\varepsilon}} c_t. \quad (13)$$

At the same time, at PFSE, the intertemporal budget constraint boils down to

$$c_t + A^{-1}c_{t+1} + \dots + A^{-n}c_{t+n} = Ak_t \quad (14)$$

By combining equations (12), (13) and (14), it is possible to obtain the explicit expression for  $c_t$  as a function of the initial endowment of capital  $k_t$ :

$$c_t = \left[ \frac{1 - [(\beta A)^{\frac{1}{\varepsilon}} A^{-1}]^n}{1 - (\beta A)^{\frac{1}{\varepsilon}} A^{-1}} + [(\beta A)^{\frac{1}{\varepsilon}} A^{-1}]^n \delta^{\frac{1}{\varepsilon}} \right]^{-1} Ak_t$$

Exploiting period  $t$  resources constraint (3), we finally obtain the explicit equilibrium law of capital accumulation associated to a degree  $n$  of myopia:

$$k_{t+1} = \gamma^{AkLF}(n) k_t$$

where

$$\gamma^{AkLF}(n) \equiv \frac{k_{t+1}}{k_t} = A \left[ 1 - \left( \frac{1 - \theta^n}{1 - \theta} + \theta^n \delta^{\frac{1}{\varepsilon}} \right)^{-1} \right] \quad (15)$$

is the (constant) balanced growth path (*BGP*) of the economy and  $\theta \equiv (\beta A)^{\frac{1}{\varepsilon}} A^{-1}$ . Notice, as one should expect, that  $\gamma^{AkLF}(n)$  is increasing in  $\delta$ , as the latter provides an incentive to invest in capital. As usual, for the *BGP* to be well defined and economically meaningful, one needs to introduce a restriction on the parameter values in order to prevent the rate of growth to be larger than the interest rate:

**Assumption 1.**  $(\beta A)^{1/\varepsilon}/A < 1$ .

It is immediate to verify that as soon as the degree  $n$  of myopia is progressively relaxed and converges to infinite, under Assumption 1 one has

$$\lim_{n \rightarrow +\infty} \gamma^{AkLF}(n) = (\beta A)^{\frac{1}{\varepsilon}} \equiv \gamma^{AkPF}.$$

It is worth noticing that  $\gamma^{AkPF}$  is the growth factor one gets in the standard *Ak* models with agents endowed with perfect foresight, as it shown, among the others, in Romer (1986) and Barro (1990). Therefore we have established a continuity criterion between economies with limited foresight and those with perfect foresight. In order to ensure the sequence of myopic economies to converge to a positive rate of growth, we must nevertheless introduce a standard condition ensuring a not too low discount factor  $\beta$  compared with the capital productivity  $A$ :

**Assumption 2.**  $(\beta A)^{1/\varepsilon} > 1$ .

At this point of the analysis, one may wonder whether, by increasing progressively the time horizon  $n$  of the individual maximization plans, i.e. by reducing the degree of myopia, one gets a sequence of economies characterized by increasing or decreasing balanced growth paths  $\gamma^{AkLF}(n)$ . The answer to this question requires calculating the derivative of  $\gamma^{AkLF}(n)$  with respect to  $n$ . In the light of its definition in (15), it is immediate to prove that the sequence  $\gamma^{AkLF}(n)$  is increasing in  $n$  if and only if  $\frac{1-\theta^n}{1-\theta} + \theta^n \delta^{\frac{1}{\varepsilon}}$  is increasing too in  $n$ . But it is not difficult to verify that this is in turn true if and only if  $\delta^{\frac{1}{\varepsilon}} (1 - \theta) < 1$ , i.e.  $\delta < (1 - \theta)^{-\varepsilon} \equiv \hat{\delta} > 1$ . As a consequence, when  $\delta < \hat{\delta}$ , by relaxing progressively the degree  $n$  of myopia, we obtain an increasing sequence of *BGP*s converging to  $\gamma^{AkPF}$ , otherwise this sequence is decreasing.

The reason at the ground of these findings is rather intuitive. If the weight  $\delta$  agents place on the utility of their last period consumption (although not implemented) is not too large, then the lengthening of the planning horizon will make it appropriate to decrease initial consumption and increase investment to finance a longer sequence of consumption streams. On the other hand, if the weight of the utility of the last period taken into consideration in the maximization problem is large enough, an extension of the planning horizon will decrease the importance of utility streams subsequent to the planning period  $t$ , and therefore discourage investment since the beginning. We can thus summarize the above results in the following Proposition.

**Proposition 2.** *In the limited foresighted Ak model with an (inverse) degree  $n$  of myopia, under Assumptions 1 and 2, one has:*

- (i) For each  $n$ ,  $\gamma^{AkLF}(n) > 0$ .
- (ii)  $\lim_{n \rightarrow +\infty} \gamma^{AkLF}(n) = (\beta A)^{\frac{1}{\varepsilon}} \equiv \gamma^{AkPF}$ .
- (iii) There exists  $\hat{\delta} = (1 - \theta)^{-\varepsilon} > 1$ , such that  $\frac{\partial \gamma^{AkLF}(n)}{\partial n} > 0$  if and only if  $\delta < \hat{\delta}$ .

An immediate corollary of Proposition 2 is that the growth factors  $\gamma^{AkLF}(n)$  of the myopic economies, for any  $n$ , are lower than the growth factor of the perfect foresight economy  $\gamma^{AkPF}$  if and only if  $\delta < \hat{\delta}$ .

Let us recall to mind that the *BGP* of the perfect foresight economy,  $\gamma^{AkPF}$ , is globally unstable. Thus the economy will grow since the beginning at this factor without displaying any transitional dynamics.

### 3 A monetary model with limited foresight

In this Section we extend the model previously analyzed by introducing *fiat* money as an asset alongside productive capital. We do not provide any rationale for money demand. As a consequence, nominal balances must be viewed as a purely speculative asset. Recall to mind, as recognized by Santos and Woodford (1997), Miao (2014) and Martin and Ventura (2018), that conditions under which bubbles exist are relatively demanding and rely on the existence of some type of financial frictions and credit market imperfections (Scheinkman and Weiss, 1986; Woodford, 1986; Le Van and Pham, 2016; Bosi et al., 2022). Such observations thus allow to better appreciate our results in terms of existence of monetary equilibria and money non-neutrality. For the purposes of our analysis, let us first discuss the monetary extension of the basic infinite horizon economy described in Section 2. The problem faced by the representative agent is to maximize (4) letting the index  $j$  scroll to infinity and subject

for each  $j$  to the budget constraint

$$M_{t+j+1}^d + p_{t+j}c_{t+j} + p_{t+j}k_{t+j+1} = p_{t+j}r_{t+j}k_{t+j} + p_{t+j}(1 - \phi)k_{t+j} + M_{t+j}^d + \tau_{t+j}$$

where  $M_{t+j}^d$  is agent's demand of money in time  $t + j$ ,  $p_{t+j}$  the price of the consumption good,  $r_{t+j}$  the real rental price of capital and  $\tau_{t+j}$  the Central Bank monetary transfers or withdrawals to the households. We assume the Central Bank to peg money growth at the constant rate  $\mu > -1$ , so as to require at equilibrium  $\tau_{t+j} = \mu M_{t+j}$  and  $M_{t+j+1} = (1 + \mu)M_{t+j}$ , where  $M_{t+j}$  denotes money supply in period  $t + j$ . Since at Walrasian equilibrium the real interest rate satisfies  $r_{t+j} = H$  and thus the gross real one  $R_{t+j} = A$ , the first order conditions for an interior solution of the maximization problem can be written as

$$c_{t+j} = \left( (A\beta)^{\frac{1}{\varepsilon}} \right)^j c_t, \quad (16)$$

and

$$p_{t+j-1}/p_{t+j} = A \quad (17)$$

for  $j = 0, 1, 2, \dots$ . Notice that the no-arbitrage condition (17) claims that the real rate of return of money holding, represented by the deflation factor, is equal to the gross return on capital investment  $A$ . By combining expressions (16) and (17) with the good market clearing condition, i.e.  $c_{t+j} + k_{t+1+j} = Ak_{t+j}$ , it is possible to show that the unique *BGP* of the economy is given by  $(\beta A)^{1/\varepsilon}$  and Assumption 1 allows for positive unbounded growth. Therefore, along such *BGP*, capital and consumption grow at the same rate:

$$\frac{k_{t+1}}{k_t} = \frac{c_{t+1}}{c_t} = (\beta A)^{\frac{1}{\varepsilon}}$$

while the dynamics of real balances  $m_t = M_t/p_t$  evolves according to

$$\frac{m_{t+1}}{m_t} = A(1 + \mu).$$

Such competitive equilibrium in order to be consistent with individual optimization behavior, requires the money growth rate  $\mu$  to be negative in order to prevent agents from assets over-accumulation or large enough indebtedness. Such condition is indeed guaranteed by the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t m_0 (A(1 + \mu))^t c_0^{-\varepsilon} (\beta A)^{-t} = 0$$

which is satisfied if and only if  $\mu < 0$ . Were this condition not respected, the value of money would require to be driven to zero, i.e., for every  $j$  one should have an infinite price level  $p_{t+j} = \infty$ . However, even when money accumulation does respect the transversality condition, money is positively valued, the dynamics of capital, consumption and *GDP* is not affected by money and is perfectly identical to that obtained in the real infinite horizon economy. As a consequence, in infinite horizon models with agents endowed with perfect foresight money is either not valued or completely neutral.

Lovo and Polemarchakis (2010) show by contrast that these properties are not longer robust to the introduction of myopic behavior. In this Section, we extend their analysis to a growth model by also accounting for monetary policy. As we will prove in the sequel, limited foresight entails indeed dramatic consequences in terms of existence of monetary equilibria, money non neutrality and equilibrium global (in)determinacy.

For a degree  $n$  of myopia, the sequence of temporal budget constraints faced by an agents maximizing (5) in time  $t$  is:

$$M_{t+j+1}^d + p_{t+j}c_{t+j} + p_{t+j}k_{t+j+1} = p_{t+j}r_{t+j}k_{t+j} + p_{t+j}(1 - \phi)k_{t+j} + M_{t+j}^d + \tau_{t+j} \quad (18)$$

for  $j = 0, \dots, n - 1$  and

$$p_{t+n}c_{t+n} = p_{t+j}r_{t+n}k_{t+n} + p_{t+n}(1 - \phi)k_{t+n} + M_{t+n}^d + \tau_{t+n}.$$

where all the variables have been previously defined. By integrating backward the budget constraint (18), and after some straightforward computations, one can write the intertemporal budget constraint as:

$$p_t c_t + \dots + p_{t+n} c_{t+n} + p_t k_{t+1} + \dots + p_{t+n-1} k_{t+n} = p_t [r_t + (1 - \phi)] k_t + \dots + p_{t+n} [r_{t+n} + (1 - \phi)] k_{t+n} + M_t^d + \tau_t + \dots + \tau_{t+n}. \quad (19)$$

The individual program can be thus reformulated as to maximize (5) subject to (19). Solving it, again requires to introduce the Lagrangian and to compute the first order conditions:

$$c_{t+j-1}^{-\varepsilon} = \frac{p_{t+j-1}}{p_{t+j}} \beta c_{t+j}^{-\varepsilon}, \quad (20)$$

$$c_{t+n-1}^{-\varepsilon} = \frac{p_{t+n-1}}{p_{t+n}} \beta \delta c_{t+n}^{-\varepsilon} \quad (21)$$

and

$$p_{t+j-1}/p_{t+j} = r_{t+j} + (1 - \phi) \quad (22)$$

for  $j = 1, \dots, n - 1$ , together with the budget constraint (19). Notice that, in view of (22), the budget constraint (19) boils down to

$$p_t c_t + \dots + p_{t+n} c_{t+n} = p_t [r_t + (1 - \phi)] k_t + M_t^d + \tau_t + \dots + \tau_{t+n}. \quad (23)$$

As in the previous Section, we now introduce the PFSE.

**Definition 3.** *A perfect foresight spot equilibrium (PFSE) for the economy is a sequence of consumption prices and rental prices of capital  $\{p_i, r_i\}_{i=0}^{\infty}$  such that, in every date  $t$ , the spot markets for capital, consumption and money balances are in equilibrium:*

$$r_i = A - (1 - \phi), \quad p_i/p_{i+1} = r_{i+1} + (1 - \phi), \quad k_{i+1} = (r_i + (1 - \phi)) k_i - c_i, \quad M_i = M_i^d \quad \text{and} \quad \tau_i = \mu M_i. \quad (24)$$

From equilibrium conditions (24), it is possible to rewrite the first order conditions in the form

$$c_{t+j} = \left( (A\beta)^{\frac{1}{\varepsilon}} \right)^j c_t, \quad (25)$$

$j = 0, \dots, n - 1$ , and

$$c_{t+n} = \left( (A\beta)^n \right)^{\frac{1}{\varepsilon}} \delta^{\frac{1}{\varepsilon}} c_t \quad (26)$$

while the intertemporal budget constraint boils down to:

$$c_t + A^{-1} c_{t+1} + \dots + A^{-n} c_{t+n} = A k_t + m_t (1 + \mu)^{n+1}. \quad (27)$$

By combining (25), (26) and (27), we thus obtain the explicit expression for the initial consumption  $c_t$  as a function of the initial stock of capital  $k_t$  and initial real balances  $m_t$ :

$$c_t = \left[ A k_t + m_t (1 + \mu)^{n+1} \right] \left( \frac{1 - \theta^n}{1 - \theta} + \theta^n \delta^{\frac{1}{\varepsilon}} \right)^{-1} \quad (28)$$

By plugging (28) into the resource constraint  $k_{t+1} = A k_t - c_t$ , we can write the equilibrium law of capital accumulation in time  $t$ :

$$k_{t+1} = A k_t - \left[ A k_t + m_t (1 + \mu)^{n+1} \right] \left( \frac{1 - \theta^n}{1 - \theta} + \theta^n \delta^{\frac{1}{\varepsilon}} \right)^{-1} \quad (29)$$

Beside equation (29) describing capital accumulation, we have the equilibrium dynamics of real balances:

$$m_{t+1} = A (1 + \mu) m_t \quad (30)$$

A *PFSE* with a degree  $n$  of myopia is thus fully described by a sequence  $\{k_t, m_t\}_{t=0}^{\infty}$  satisfying equations (29) and (30) in each time  $t$  and subject to the initial condition  $k_0 > 0$  and  $M_0$ . Since equation (30) is autonomous in real balances, by defining  $x_t \equiv k_t/m_t$  the capital to real balances ratio, we can express equation (29) as:

$$A(1 + \mu)x_{t+1} = Ax_t - \left[ Ax_t + (1 + \mu)^{n+1} \right] \left( \frac{1 - \theta^n}{1 - \theta} + \theta^n \delta^{\frac{1}{\varepsilon}} \right)^{-1} \quad (31)$$

Notice that (31) represents a first-order linear difference equation in the capital to real balances ratio  $x$ . It follows that equation (31) possesses a unique interior stationary solution whose stability analysis will provide us with useful insights in terms of equilibrium global (in)determinacy. The explicit expression of the interior stationary solution of (31) is found as

$$x = -\frac{1}{A} \frac{(1 + \mu)^{n+1} \left( \frac{1 - \theta^n}{1 - \theta} + \theta^n \delta^{\frac{1}{\varepsilon}} \right)^{-1}}{\mu + \left( \frac{1 - \theta^n}{1 - \theta} + \theta^n \delta^{\frac{1}{\varepsilon}} \right)^{-1}} \quad (32)$$

One thus verifies that the stationary capital to real money balances ratio  $x$  defined in (32) is positive if and only if  $-1 < \mu < -\left( \frac{1 - \theta^n}{1 - \theta} + \theta^n \delta^{\frac{1}{\varepsilon}} \right)^{-1}$ . In addition, for any stationary capital to real money balances ratio  $x$ , the (common) balanced growth path  $\gamma^{MoneyLF}$  for  $k_t$ ,  $c_t$  and  $m_t$  is immediately obtained by substituting (32) into (29):

$$\gamma^{MoneyLF} = (1 + \mu)A. \quad (33)$$

Introducing limited foresight thus not only invalidates the money neutrality feature typical of infinite horizon models, but also entrusts to the money growth rate the target to drive the rate of growth of the whole system. More specifically, a monetary long-run equilibrium is a *BGP* where the economy grows at the same rate as real balances. Observe that real balances growth rate is increasing in capital productivity  $A$  and in the rate  $\mu$  at which the Central Bank increases or withdraws money balances. In the opposite case of a non monetary long-run equilibrium, the economy grows at a different rate with respect to that of real balances, as is the case in infinite horizon economies.

**Definition 4.** *A monetary long-run equilibrium is a BGP where the economy grows at the same rate as real balances.*

After having provided the explicit expression for the stationary capital to real balances ratio  $x$ , we must ensure that the former is compatible with positive consumption. Since alongside the *BGP* one has  $c/k = -\mu A$ , it follows that non-negative consumption requires  $\mu \leq 0$ . In addition, it is easy to verify that  $x$  is positive, and thus real balances are positive

too, if and only if  $-1 < \mu < -\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta \frac{1}{\varepsilon}\right)^{-1}$ , being negative otherwise.<sup>2</sup> It is also immediate to verify that when  $\mu = 0$ , we obtain negative monetary equilibria, while Lovo and Polemarchakis (2010) find also positive ones. This apparent contradiction is easily explained by observing that Lovo and Polemarchakis (2010) introduce neoclassical production functions with factor substitutability and where capital can thus adjust to restore the arbitrage condition  $f'(k) = 1$ . In our simple  $Ak$  framework, instead, the marginal productivity of capital is bound to remain constant and thus any adjustment in capital to implement the golden rule is not made possible. Eventually, let observe that, since  $\mu < 0$ , one has  $\lim_{n \rightarrow \infty} x = 0$ , i.e. as soon as myopia is completely removed, the monetary equilibrium disappears.

The reason underlying the existence of a monetary equilibrium with short-sighted agents only under a monetary contraction must be sought starting from the analysis of the budget constraint (29). In this constraint, in fact, it is possible to see how among the resources, alongside the discounted capital income  $Ak_t$ , total monetary income also appears, i.e.  $m_t(1 + \mu)^{n+1}$ . Since this flow is actualized only for a finite number  $n$  of periods, it is bounded and therefore affects consumption and investment plans, as shown in (28) and (30). Relaxing the hypothesis of myopia, i.e. increasing  $n$ , in the case  $\mu \leq 0$ , as we assume in the paper, monetary income would decrease and finally converge to zero. By contrast, when  $\mu > 0$ , the monetary income would increase and finally explodes as soon as  $n$  becomes larger and larger. At this point, the maximization problem would not be well defined. On the other hand, in the case of equilibrium with negative money, as  $n$  is progressively increased, monetary income would tend to  $-\infty$ , and there would obviously not be any feasible solution for the maximization problem.

We have proved that system described by (31) possesses a unique stationary equilibrium, which can be associated with positive as well as negative money. Our goal is now to study the stability of such stationary solution. In view of the linearity of (31), such analysis turns out to be reliable not only locally but also from a global perspective. In addition, since the capital to real balances ratio  $x = k/m$  in each time  $t$  depends upon the current price level, it represents a non predetermined variable. Such observation entails important consequences in terms of local and global indeterminacy. To this end, the Jacobian  $J$  of equation (31) is immediately computed as:

$$\frac{dx_{t+1}}{dx_t} = \frac{1 - \left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta \frac{1}{\varepsilon}\right)^{-1}}{1 + \mu}. \quad (34)$$

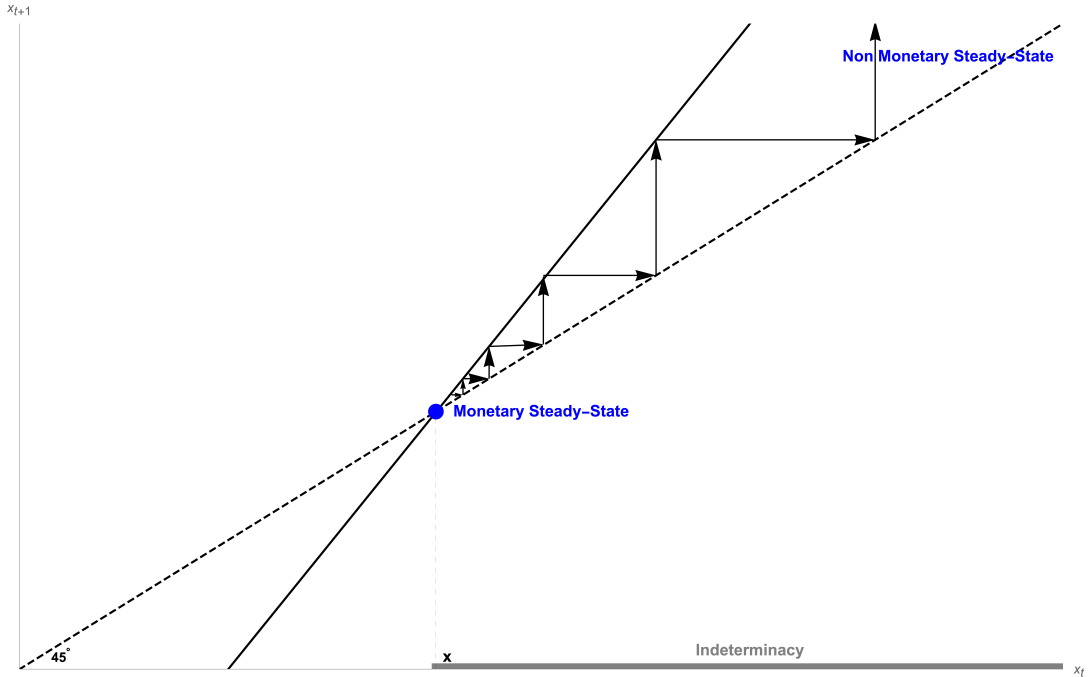
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<sup>2</sup>Notice that  $-\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta \frac{1}{\varepsilon}\right)^{-1}$  is increasing in  $\delta$  et goes to zero when  $\delta \rightarrow +\infty$ : only monetary equilibria with positive money exist.



It is immediately verifiable that  $J$  is always definite positive. More in details,  $J$  is larger than one for  $-1 < \mu < -\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta^{\frac{1}{\varepsilon}}\right)^{-1}$  and lower otherwise. Hence, the steady state associated to positive money is globally unstable, while that associated to negative money is globally stable. The instability of the positive monetary steady state means that the unique way to implement it requires the capital to real money initial ratio  $x_0$  to jump since the beginning on its fixed point  $x$ . This in turn requires the price level to adjust in time zero to solve  $p_0 = (M_0/k_0)x$ . The immediate consequence of such findings is that any positive monetary steady state is locally determinate. However, any alternative sequence for the capital to real balances ratio starting from some  $x_0 > x$ , will generate an explosive dynamics diverging to infinite and thus making the economy to converge toward the non monetary stationary equilibrium. Such equilibrium is characterized, in the light of equation (29), by the same *BGP*, given by  $\gamma^{AkLF}(n)$  in equation (15), obtained in the real limited foresight model. Notice that any initial condition  $x_0 < x$  generates a dynamics such that, after some period, all the real variables become negative and thus is not consistent with equilibrium. The global dynamics around the positive monetary equilibrium is depicted in Figure 1.

Figure 1:  $-1 < \mu < -\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta^{\frac{1}{\varepsilon}}\right)^{-1}$

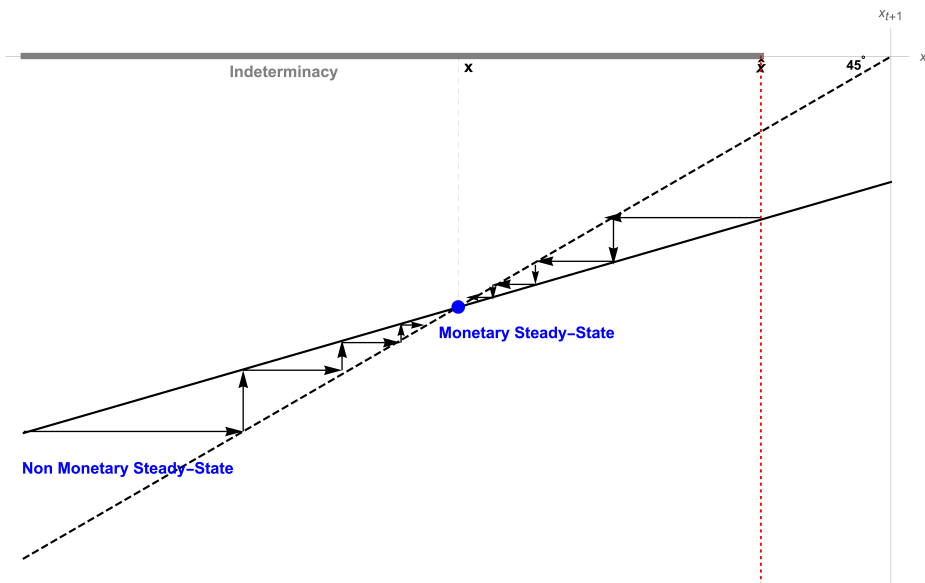


When  $\mu$  falls conversely within the interval  $\left(-1, -\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta^{\frac{1}{\varepsilon}}\right)^{-1}\right)$ , we have a negative monetary equilibrium. In such configuration  $J$  defined by (34) is lower than one. As a consequence, the interior steady state  $x$  is stable and thus globally indeterminate. Accordingly, there will exist infinite many initial price levels  $p_0$  consistent with the convergence toward it.

On the other hand, the non-monetary equilibrium  $x = \infty$  will now be unstable, i.e. locally determinate and its implementation will require since the beginning an infinite price level.

The global dynamics of the positive monetary equilibrium is depicted in Figure 2, where the red vertical line delimits the interval of value of  $x_t < \hat{x}$  compatible with positive initial consumption.

Figure 2:  $-\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta \frac{1}{\varepsilon}\right)^{-1} < \mu \leq 0$



All the results above introduced can be summarized in the following Proposition:

**Proposition 5.** *In the limited foresighted monetary Ak model, one has*

(i) *A monetary equilibrium exists if and only if the money growth rate is non positive.*

(ii) *When  $-1 < \mu < -\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta \frac{1}{\varepsilon}\right)^{-1}$  a unique, globally unstable, positive monetary interior steady state exists such that the growth factor of the economy is  $\gamma^{MoneyLF} = (1 + \mu)A$ . In addition, there exists a non monetary steady state globally stable, and thus globally indeterminate, associated to the BGP  $\gamma^{AkLF} = A \left[1 - \left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta \frac{1}{\varepsilon}\right)^{-1}\right]$ .*

(iii) *When  $-1 < \mu < -\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta \frac{1}{\varepsilon}\right)^{-1}$  a unique, globally stable, and thus globally indeterminate, negative monetary interior steady state exists such that the growth factor of the economy is  $\gamma^{MoneyLF} = (1 + \mu)A$ . In addition, there exists a non monetary steady state globally unstable, and thus globally determinate, associated to the BGP  $\gamma^{AkLF} = A \left[1 - \left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta \frac{1}{\varepsilon}\right)^{-1}\right]$ .*

We have obtained the growth rate in the real and monetary economies both under the hypothesis of perfect foresight and of limited foresight. One may wonder at this point if it is possible to rank such growth rates. We now provide a complete comparison of the growth

rates obtained under the different hypotheses we have made in order to derive important monetary policy implications.

### 3.1 Comparing growth rates

In the previous Sections, we have provided the explicit expressions for the *BGP*s of the infinite horizon economy ( $\gamma^{AkPF}$ ), and of the real and monetary economies with limited foresight (respectively,  $\gamma^{AkLF}$  and  $\gamma^{MoneyLF}$ ). We now wonder if it is possible to rank such *BGP*s, recalling nevertheless to mind that larger growth rates do not imply necessarily a Pareto improvement, since they can entail capital over-accumulation phenomena. In order to rank the different *BGP*s, let us observe that only  $\gamma^{AkLF}$  depends upon the preference parameter  $\delta$  and the degree  $n$  of myopia, while the rate  $\mu$  of money growth influences exclusively  $\gamma^{MoneyLF}$ . In addition, we have proved in Section 2 that  $\gamma^{AkLF} < \gamma^{AkPF}$  if and only if  $\delta < \hat{\delta}$ . In addition, it is easy to prove that  $\gamma^{MoneyLF} < \gamma^{AkLF}$  when  $\mu \in \left(-1, -\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta^{\frac{1}{\varepsilon}}\right)^{-1}\right)$ , being  $\gamma^{AkLF} < \gamma^{MoneyLF}$  otherwise. Finally,  $\gamma^{MoneyLF} < \gamma^{AkPF}$  for  $\mu < \theta - 1$ , and  $\gamma^{AkPF} < \gamma^{MoneyLF}$  for  $\theta - 1 < \mu \leq 0$ , being  $\gamma^{MoneyLF} = \gamma^{AkPF}$  at  $\mu = \theta - 1$ . All these pieces of information allow to prove the following Proposition:

**Proposition 6.** *Assume  $\delta < \hat{\delta} = (1 - \theta)^{-\varepsilon}$ . Then:*

(i) *When  $\mu \in \left(-1, -\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta^{\frac{1}{\varepsilon}}\right)^{-1}\right)$ , one has  $\gamma^{MoneyLF} < \gamma^{AkLF} < \gamma^{AkPF}$ .*

(ii) *When  $\mu \in \left(-\left(\frac{1-\theta^n}{1-\theta} + \theta^n \delta^{\frac{1}{\varepsilon}}\right)^{-1}, \theta - 1\right)$ , one  $\gamma^{AkLF} < \gamma^{MoneyLF} < \gamma^{AkPF}$ .*

(iii) *When  $\mu \in [\theta - 1, 0)$ , one has  $\gamma^{AkLF} < \gamma^{AkPF} \leq \gamma^{MoneyLF}$ .*

*Assume  $\delta > \hat{\delta} = (1 - \theta)^{-\varepsilon}$ . Then:*

(iv) *When  $\mu \in (-1, \theta - 1]$ , one has  $\gamma^{MoneyLF} \leq \gamma^{AkPF} < \gamma^{AkLF}$ .*

(v) *When  $\mu \in (\theta - 1, 0)$ , one has  $\gamma^{AkPF} < \gamma^{AkLF} < \gamma^{MoneyLF}$ .*

Proposition 6 shows in particular that the monetary authority can implement the *BGP* of the perfect foresight model, namely  $\gamma^{AkPF}$ , by pegging monetary growth at rate  $\mu = \theta - 1$ . In the following Section we prove that such monetary growth rate actually corresponds to the optimal monetary policy maximizing welfare.

## 4 Welfare and optimal monetary policy

We have seen that in the monetary economy with limited foresight, the growth rate evaluated along the *BGP* coincides with the rate of growth  $A(1 + \mu)$  of real balances. Since it is increasing in  $\mu$ , money is not neutral and the monetary authority can set  $\mu$  to influence welfare. To this end, recall to mind that the domain of definition of  $\mu$  is the interval  $(-1, 0)$ ,

otherwise consumption would not be positive. In this Section, we shall compute the optimal rate of growth of nominal balances maximizing social welfare. To this end, we need previously to better define the social planner target, since her planning horizon is infinite, by contrast to the households limited one. Therefore, along the monetary *BGP* the consumption plans concretely implemented (not those merely planned by myopic households) from period  $t$  up to infinite satisfy for all  $j \geq 1$

$$c_{t+j} = (A(1 + \mu))^j c_t$$

and thus the instantaneous utility is

$$u(c_{t+j}) = \frac{[(A(1 + \mu))^j c_t]^{1-\varepsilon}}{1 - \varepsilon}.$$

In addition, we have that initial consumption satisfies  $c_t = -\mu Ak_t$ . Total discounted utility can be thus written as

$$V = \frac{(Ak_t)^{1-\varepsilon}}{1 - \varepsilon} \sum_{j=0}^{\infty} (-\mu)^{1-\varepsilon} \beta^j [(A(1 + \mu))^j]^{1-\varepsilon}$$

Assuming that the sequence is convergent, i.e.  $\beta [A(1 + \mu)]^{1-\varepsilon} < 1$  (this is true for  $\varepsilon \geq 1$  and for  $\varepsilon$  not too much close to zero), we can write the social welfare in the following form:

$$V(\mu) = \frac{(Ak_t)^{1-\varepsilon}}{1 - \varepsilon} \frac{(-\mu)^{1-\varepsilon}}{1 - \beta [A(1 + \mu)]^{1-\varepsilon}}. \quad (35)$$

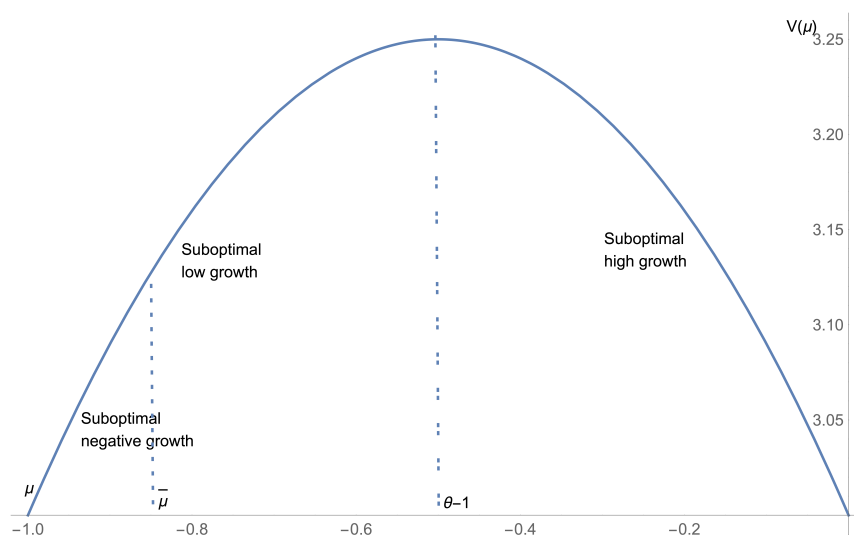
It is easy to prove that  $V(\mu)$  describes in the interval  $(-1, 0)$  a typical bell-shaped curve. It starts from some finite value at  $\mu = -1$ , then grows monotonically (indeed  $\partial V(\mu)/\partial \mu > 0$ ) and reaches a maximum at  $\mu = \theta - 1$  where its derivative vanishes. Then it decreases (its derivative being negative) up to  $\mu = 0$ . Therefore welfare is maximized in correspondence to  $\mu = \theta - 1$  that should represent the target of a benevolent monetary authority. In addition, notice that  $\mu = \theta - 1$  allows to implement exactly the growth factor  $\gamma^{AkPF}$  of the non-monetary optimal perfect foresight economy. It is not a coincidence that it corresponds to the Friedman' rule, according to which the Central Bank should withdraw money at a factor equal to the growth rate to interest rate ratio.<sup>3</sup>

In Figure 3 we have depicted (35). One observes that for  $\mu < \theta - 1$ , the economy rate of growth is too low (possibly even negative) to be efficient. On the other hand, for  $\mu > \theta - 1$ , the growth rate is too large to be optimal, since it requires a strong contraction in consumption.

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<sup>3</sup>Notice that in the stationary economy analyzed by Friedman (1969) our conditions boils down to  $\mu = \beta - 1$ , which insures the nominal interest rate to be driven to zero.

Figure 3: Welfare



## 5 Concluding remarks

We have studied both real and monetary growth models by assuming agents to be endowed with limited foresight. Accordingly, in each time agents formulate plans only for a finite number of periods. These plans are in turn revised in each subsequent date, taking the assets endowment planned in the previous period as initial conditions. We have shown that limited foresight may lead to capital under-investment and be thus growth-detrimental. However, by relaxing progressively the degree of myopia, the economy growth rate increases monotonically and converges, when myopia is completely removed, to the Perfect Foresight Equilibrium. By introducing *fiat* money as a bubble, contrary to economies with perfect foresight and free from financial market imperfections, borrowing constraints and limited participation, we obtain real balances dynamics to influence short run as well as long run equilibrium real variables. This opens the door for shaping the optimal monetary policy to maximize welfare. We individuate such policy in the Friedman rule requiring to withdraw money at a factor equal to the growth rate to interest rate ratio. We also compare and rank the *BGP*s obtained in the real and monetary economies with limited foresight with that corresponding to *PFE*. We have also performed a global stability analysis of monetary equilibria and proved that when money is positive, the stationary capital to real balances ratio is globally unstable; however, there arises an alternative and globally stable stationary non-monetary equilibrium. On the other hand, when money is negative, one gets a globally stable monetary equilibrium and a globally unstable non-monetary one. Whatever the configuration, global indeterminacy is thus bound to prevail.

The hypothesis of limited foresight can be extended to a large number of intertemporal models. As an example, instead of framing money as a pure speculative bubble, one can motivate its demand on the ground of the liquidity services it provides and introduce a cash-in-advance constraint on real expenditures (Abel, 1985; Bosi et al., 2005) or opt for a money-in-utility function approach (Sidrauski, 1967). Our monetary framework can also be improved by introducing financial frictions (Bosi et al., 2022) or borrowing constraints (Woodford, 1986; Grandmont et al., 1998). Also a direct comparison with *OLG* models characterized by market limited participation could be performed, in the spirit of Lovo and Polemarchakis (2010), although this would require to account also for labor income beside capital one. In our models we have assumed a simple monetary policy such that the Central Bank pegs the money growth rate; a fruitful extension would be to account for alternative monetary rules, as the Taylor ones aimed at pegging the nominal interest in reaction to some objective gap. Such analysis may be carried out within New Keynesian sticky prices models (Schmitt-Grohé and Uribe, 2004; Magris and Onori, 2024). Finally, the hypothesis of limited foresight can be usefully coupled with uncertainty and, even more interesting, with structural and unpredictable change, as the environmental and climate ones, in respect to which standard *DSGE* models seems to be only poorly performing. We leave such considerations for future research.

## References

- Abel, A. B. (1985). Dynamic behavior of capital accumulation in a cash-in-advance model. *Journal of Monetary Economics*, 16(1):55–71.
- Barro, R. J. (1990). Government spending in a simple model of endogenous growth. *Journal of Political Economy*, 98(5, Part 2):S103–S125.
- Bosi, S., Le Van, C., and Pham, N.-S. (2022). Real indeterminacy and dynamics of asset price bubbles in general equilibrium. *Journal of Mathematical Economics*, 100:102651.
- Bosi, S., Magris, F., and Venditti, A. (2005). Multiple equilibria in a cash-in-advance two-sector economy. *International Journal of Economic Theory*, 1(2):131–149.
- Evans, G. W. and Honkapohja, S. (2012). Learning and expectations in macroeconomics. In *Learning and Expectations in Macroeconomics*. Princeton University Press.
- Friedman, M. (1969). *The Optimum Quantity of Money*. Chicago: Aldine Publishing Co.
- Gabaix, X. (2020). A behavioral new keynesian model. *American Economic Review*, 110(8):2271–2327.
- Grandmont, J.-M., Pintus, P., and De Vilder, R. (1998). Capital–labor substitution and competitive nonlinear endogenous business cycles. *Journal of Economic Theory*, 80(1):14–59.
- Jones, L. E. and Manuelli, R. E. (2005). Neoclassical models of endogenous growth: The effects of fiscal policy, innovation and fluctuations. *Handbook of economic growth*, 1:13–65.
- Le Van, C. and Pham, N.-S. (2016). Intertemporal equilibrium with financial asset and physical capital. *Economic Theory*, 62(1):155–199.
- Lovo, S. and Polemarchakis, H. (2010). Myopia and monetary equilibria. *Journal of Mathematical Economics*, 46(5):925–936.
- Lucas, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22(1):3–42.
- Magris, F. and Onori, D. (2024). Taylor and fiscal rules: When do they stabilize the economy? *Mathematical Social Sciences*, 128:68–89.
- Martin, A. and Ventura, J. (2018). The macroeconomics of rational bubbles: a user’s guide. *Annual Review of Economics*, 10(1):505–539.

- Miao, J. (2014). Introduction to economic theory of bubbles. *Journal of Mathematical Economics*, 53:130–136.
- Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, 94(5):1002–1037.
- Santos, M. S. and Woodford, M. (1997). Rational asset pricing bubbles. *Econometrica: Journal of the Econometric Society*, pages 19–57.
- Scheinkman, J. A. and Weiss, L. (1986). Borrowing constraints and aggregate economic activity. *Econometrica: Journal of the Econometric Society*, pages 23–45.
- Schmitt-Grohé, S. and Uribe, M. (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, 114(2):198–230.
- Sidrauski, M. (1967). Inflation and economic growth. *Journal of Political Economy*, 75(6):796–810.
- Woodford, M. (1986). Stationary sunspot equilibria in a finance constrained economy. *Journal of Economic Theory*, 40(1):128–137.
- Woodford, M. (2019). Monetary policy analysis when planning horizons are finite. *NBER Macroeconomics Annual*, 33(1):1–50.
- Woodford, M. and Xie, Y. (2022). Fiscal and monetary stabilization policy at the zero lower bound: Consequences of limited foresight. *Journal of Monetary Economics*, 125:18–35.